OUR IRONCLADS
AND
MERCHANT SHIPS

With the Authors Compliments

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AND

MERCHANT SHIPS.

BY

REAR-ADMIRAL

E. GARDINER FISHBOURNE, C.B.

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TO THE OFFICERS

OF THE

ROYAL NAVAL AND MERCANTILE MARINE

SERVICES

AND TO

NAVAL ARCHITECTS AND SHIPOWNERS,

THIS WORK ON

"OUR IRONCLADS AND MERCHANT SHIPS"

IS RESPECTFULLY DEDICATED

BY

REAR-ADMIRAL E. GARDINER FISHBOURNE, C.B.

ERRATA.

Page x, line 9, for "vertical," read virtual.

" 14, instead of $= 2 = 2$, read $2 + 2$.

" 42, Art. 132, for "so," read dangerous.

" 44, " 137, read relatively "*raised.*"

" 79, for $Mr = 6$ feet, read $Mr = 9$ feet.

" 82, for the portion of the arcs, read the position of the arch.

" 85, read $V = \left(\frac{dx^2 + dy^2 + dz^2}{dt^2}\right)^{\frac{1}{2}}$.

" 87, for Fig. XXII, read Fig. XVIII; and for "put OF, Fig. XX, $= 9$," read put OF, Fig. XX, $= 9$.

" 88, for $\Sigma = f\,(y_2^2 + z_2^2)$, read $(\Sigma = f\,(y_2^2 + z_2^2)\,dM$.

" 96, 4th paragraph, for "low," read law.

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PREFACE

TO

ENLARGED EDITION OF

"OUR IRONCLADS AND MERCHANT SHIPS."

WHEN we have deducted the misrepresentations of us by our opponents, their contradictions of each other, and of themselves, together with their oppositions of science falsely so-called, there is but little left for us to notice.

In this edition we have continued our mathematical investigations leading to demonstrations of the laws which govern the motions of ships at sea.

Some have affected, to have discovered to our discomfiture, that our method of calculating stabilities from the external pressures on the surface of the body, yields the same result as the system now in use. But these have been unable to discern that all the arguments in our book proceed, for the present, on the hypothesis of the theoretical correctness of the old system.

We have introduced in an Appendix, page 107, a fuller statement as to the erroneous character of the Froude theory and of the dangers arising from Mr. E. J. Reed's application of that theory.

In various publications issued long before Mr. Froude was known to the public, we indicated the cause of greater or less rolling of ships, in a comparison of the "Canopus" with "Vanguard" and "Vanguard" with "Superb."

The cross sections of "Canopus" were of a U-form, while those of "Vanguard" were of a V-form with greater proportionate breadth. "Canopus" had much ballast, and other weights low down, including weight of hull; therefore her centre of gravity low, and weights laterally concentrated.

"Vanguard," on the contrary, had little ballast her weights including that of hull thrown up and out, therefore her centre of gravity was very high, and her weights were laterally more distributed.

"Vanguard" possessed vastly more stability under canvas, which was derived from her much greater breadth.

The "Canopus" was easy, her rolling was slow and limited. The rolling of the "Vanguard" was specially uneasy, and frequently it was frightfully great and rapid.

We pointed out that no altered disposition of weights could cure the "Vanguard's" defects; for, if weight was raised the arcs of roll would be increased, but those were already too great; while, if the weight was lowered, her rolling would be made more rapid, but that was already too rapid.

This condition arose much from a radical defect in her form owing to which, as she rolled over and back, the centre of gravity rose and fell considerably. The acceleration from that fall increased the arcs and rapidity of roll, intensifying the evils of two great breadth of beam.

The "Vanguard" and "Superb" were sister ships, the latter was a little longer at fore foot, which brought her centre of gravity of hull lower, she had more ballast and was immersed one foot more than the "Vanguard," by water provisions, &c., therefore had a lower centre of gravity, yet she was easier, rolled slower, and through smaller arcs, and was a faster and better ship in every way.

We fear that notwithstanding this dear-bought experience, in the haste to escape the dangers of the Froude-Reed "Scylla" of *no* stability, we are running into the "Carybdis" of undue stability from excessive breadth of beam, from which our ships will have motions only less great than those of the "Vanguard," because the cross sections of the modern ships are better than theirs.

In the very numerous "Vanguard" forms we had frightfully extensive and rapid rolling, partly resulting from a condition similar to that proposed by Mr. Froude as a panacea for the evils of rolling, increased inertia of sides from distributed weights, and a high centre of gravity.

The effect of the distribution of weight on the sides is to reduce the stability by depriving the sides of their buoyant or supporting power. The effect of raising the centre of gravity is to reduce at once the stability and the inertia which our opponents desire to increase.

Our challenge to produce an authentic case of a ship in which the arcs rolled through have been reduced by raising the centre of gravity, has been four years before the public without an answer.

It is contrary to the nature of things that such a case could exist, and, therefore, it was not found.

We are glad to find from "Naval Science," April No., 1874, that the Rev. Dr. Woolley, Mr. E. J. Reed, and Mr. Froude himself are now "*fully impressed with the necessity of caution in the application of his (Mr. Froude's) theory.*

Two of these gentlemen were members of the Scientific Committee on the designs for ships; and we gathered that they had had doubts of the correctness of this system, when we found them recommending *great* stability (notwithstanding the evils they anticipated) in opposition to the little or no stability that they had previously counselled.

This is one of those cases where common sense and experience have been divorced from mathematics, and the latter still called an exact science.

If the plan proposed by Mr. Froude, that of keeping ships upright by placing great weights on their sides and raising their centres of gravity, so that waves shall pass without rolling them, had been arrived at, what then? Why, in proportion as the waves increased in height, they would roll over the bulwarks, sweep the decks of everything, and fill the ship, if not closely battened down, if the weight of the body of water did not previously capsize her.

Reducing the stability would facilitate a capsize—1st. From the large body of water alluded to; 2nd. From inability to resist squalls of wind or the impact of a heavy wave; 3rd From the accumulation of water when the bulwarks were high.

Then from deficiency of stability the sails would be ineffective; steering would be difficult and uncertain, and the ship would be so easily inclined that her guns would frequently be useless.

In a word, the system has not one redeeming point, as may, perhaps, be admitted when a few more lives have been sacrificed. Yet the system has been approved by those said to be the greatest authorities on naval science.

Nevertheless the ships operated on were inclined to roll over, till from 350 to 500 tons of ballast were placed between the double bottoms of each; yet this, according to the theory, should have made them more liable to roll over. They were found to roll frightfully, and long and heavy iron bilge pieces were placed on some of their bottoms, adding thereby the most effective ballast. By the double action of this weight and friction their rolling has been much reduced.

After years of ridicule it is now admitted in "Naval Science, 1874," that we have proved—pages 34 to 40, and 62—64 of this work, that an unequal distribution of weights and buoyancies, without any change in the height of the centre of gravity, produces radical changes proportionate to the extent of these inequalities,—changes not previously admitted or allowed for,—therefore, that all calculations hitherto made for the purpose of determining the amount of stability were erroneous.

Consequently, all ships with loaded sides and empty cellular bottoms, where these inequalities are the greatest, are dangerously deficient in stability as compared with the amounts assigned by former calculations.

It is now said that the amount and range of stability, *not has been*, but "*can* in all cases be easily calculated by the common rules, and then it may be *inferred* whether the stability is sufficient." Thus the knowledge indispensable for safety of life and property is proposed to be left to an *inference*.

We cannot trust to such where we ought to have proof, particularly when *inferences* have proved to be so deceptive.

The fact is, this is an unwilling admission as to another grave defect in the "common rules," which have been assumed to be always strictly accurate and accurately applied, whereas they have not been even approximately so, especially, in the case of ironclads.

We have added demonstrations of additional problems. The objects of these and the further mathematical investigations are, to demonstrate that a ship does not in any case revolve round her centre of gravity, and to determine the point or line round which she does turn, and to determine the motions, oscillations, and times of roll, arising from any applied forces.

The geometrical discussions are designed to evolve principles for facilitating the above.

Our readers must, in some few cases, take for granted for the present the correctness of the processes, though not given in detail for shortness, by which results are arrived at, such, for instance, as those at page 88:—

$$\int x_2 y_2 dM = -\frac{1}{2u_1}\left(\frac{d\Sigma}{d\theta}\right);$$

And—

$$\int x_2 z_2 dM = -\frac{1}{2}\left(\frac{d\Sigma}{d\phi}\right).$$

A result may be arrived at by a course of reasoning, true in itself, but it may land us in an equation which is not soluble by any generally known calculus, therefore we sometimes are obliged to have recourse to a more powerful calculus, nevertheless such as has been accepted by distinguished mathematicians; added to which the results prove the correctness of the process.

Reasons a new system is proposed:—

1. The old system or common rules being deduced from formulæ framed on the supposition that the inclination would be only to an infinitely small angle, while, in fact, the ships incline to large angles.

2. Proceeding on the supposition that the immersed bodies of ships were homogeneous, though they never are so.

3. Proceeding on the supposition that ships always rotate round their centres of gravity, which is never the case, therefore it is impossible to obtain accuracy by such procedure, nor were it more accurate could it yield the full results obtainable from a system of calculations deducible from the pressures on the external surfaces as proposed.

Eventually it will be seen that the longer methods which in some cases have been adopted by us, are necessary.

Until the Froude-Reed designs were introduced into the navy, our architects would have thought themselves foolish or just subjects for a criminal prosecution if they gave a ship so small an amount of metacentric height as 3 feet,* as it had been uniformly found that such ships had proved to be crank and dangerous, and had been characteristically called coffins, as so many of the smaller vessels with a small measure of stability had capsized.

Since then we have had ships with 2·48 feet, 2·2 feet, 1·8 feet, and 1·5 feet of nominal metacentric height in the "Invincible" class, but in truth much less, in consequence of their loaded topsides and empty bottoms. These also have justified the above judgment and the experience of ages, for a fleet of these men-of-war and transports have been pronounced "unseaworthy" by those who had a share in the responsibility of their designs; if not all in this very language, yet they condemned all either to receive a quantity of ballast or have their masts and sails reduced, or both.

What other could have been expected when the tried officers of the Constructive Department, the possessors of the traditional experience of ages, were cast out to make room for a man without practical knowledge? It would be instructive to place on record at the Admiralty the amount this freak has cost the country.

* The "Achilles" appears to be an exception to this, but it is not so; her great size and small masts mask the defect in fine weather; in a seaway she rolls much: and, similarly, "Minotaur," with a height of 3·8 feet, rolled 39°, when the small and less well formed wooden "Topaze" rolled only 22°.

Some years since, we suggested to the Admiralty that a table should be prepared, for their Lordships' protection against wild schemes, which should contain the dimensions of one of each of the typical ships, together with the calculated results of the various elements, including the metacentric heights, so that similar should be calculated for any new design, and on interpolation of these the true value of the design in its essential features might be found.

Had such been prepared, and had the "Invincible," "Sultan," and other designs been thus tested, they would have found a place amongst the coffins. It would well repay the cost to have such a table prepared now, in which to contrast to the Froude-Reed ships with those French and English that are found to be safe and useful.

That a greater number of catastrophies from the use of the upside down system have not occurred, has been much because the danger was discovered in good time, and much because their masts have been small, and the general mode of locomotion by steam.

In the Appendix 107 we have shown that nothing but evil could be expected from the introduction of this system.

We seem to have arrived at a climax of absurdity with respect to this practice of building ships bottom upwards. We are informed in "*Naval Science*" "that the stability of the same vessel may be greater if placed bottom upwards and range over a large angular space may be allowed, as is indeed the case with most ships; and yet the vessel may be perfectly safe when floating upright, if the conditions above indicated be complied with."

The language is Delphic but as some ideas struggle through we will deal with them.

It may be admitted that ships built bottom upwards may be perfectly safe when *floating upright*, but we must add a further condition, if there is some means of keeping them upright, but seeing that the Froude-Reed system has failed to effect this and none other is offered, we must accept the reluctant admission that such ships are *not safe*, and we find that at the first temptation they are disposed to resent the violence that has been done them in trying to prevent their following the natural course.

The editor seems clearly to have had further misgivings about what he had written for, he adds another limitation, "*if the conditions above indicated be complied with.*"

He cannot mean by "above indicated" to strip a section off the side of a ship, and thus *perforce* capsize her as *Naval Science* well illustrated. He must, therefore, mean what he is reluctant more clearly to express, the conditions are those which he has admitted we have proved to be correct and necessary for safety. Pages 34 to 44 of this book.

But those conditions require that ships should not be built bottom upwards, but that buoyancy should be placed in its natural position *above,* and weight in its equally natural position *below,* for if not they will take the first opportunity to change places, all consequences of emptying the crew and cargo into the sea notwithstanding, this much is so undesirable that we need not follow them further. But we feel bound to draw attention to the grave responsibility that is incurred by those who permit the construction of ships on a plan that is confessed to be unsafe by one of its originators who has had such ample means of obtaining this knowledge; and by others, its parent included, who now admit that the system is only true within the limits stated by J. C.

We have said that a ship never rotates round her centre of gravity; we have given a proof of this, page 142 to 145, that may be understood by any person of ordinary intelligence and instruction.

A ship is always under constraint from the water which floats her, and her motion may be compared to that of a bale of goods, the centre of gravity of which is at its central point, and which a porter desires to place upon a truck; using the edge of the truck as a fulcrum, he first raises one end, then the other; in neither process is the body turned round its central point or centre of gravity; neither is a ship, for her centre of gravity, as she is rolled or is inclined over, rises or falls, the water being the fulcrum in this case, the body and centre of gravity rotating round some point that neither rises or falls, but generally continues in the water-line.

It is clear then that the leverage of all the forces and moments must be measured from this latter point rather than from the centre of gravity, which not only moves with each motion of the ship vertically, but more, does not continue at the same distance from the point round which it turns.

The locus of each point can be determined in all cases.

All deductions, therefore, made on the supposition that a ship revolves round her centre of gravity, must be erroneous, sometimes grossly so. In reality, the metacentre and equilitrating lever have no existence in fact.

True, we appear to show, Figure XIII, while reasoning on one of our blocks, that the deductions from the old system to agree with those from the system of taking the external pressures. In this case they agree, but that is the only case in which they do. 1st. Because the body is homogeneous, and of such specific gravity and form that the centre of gravity always remains in the water-line, however so much may be the inclination; and 2ndly, because the centre of gravity of displacement was there accurately determined.

Simple calculations from the external pressures will always define strictly the position of the centre of gravity of displacement when it is necessary to obtain it; in general it cannot be determined accurately by the old system, and never without more elaborate calculations.

But the fact is, all calculations made on an equilibrating lever or metacentric system, are dangerously defective, and the attempts to estimate stabilities and rolling motions in a seaway by such means are still more delusive; for at best, such could only give an estimate of the statical effect, whereas rolling motions are for the most part the result of dynamic action, and this is specially so when the waves are travelling with great velocity.

We have proved, by the principle of *vertical velocities*, at it is absurd to seek in the conditions under which the floating body would assume a state of rest or equilibrium through the action of those pressures and the *equilibrating lever*, which is without prop or support.

On the other hand, we have proved mathematically that the external and downward pressures of the body may be combined, according to the principle of D'Alembert, to determine the extent and circumstances of the motions of the floating body. And further, we have established, by the principle of vertical velocities, the conditions under which the body assumes a state of rest or of equilibrium, in any given position, by the action of the external forces and the downward pressure of the body.

Now, while an estimate of the dynamic action is not attempted under the metacentric method, the effect can be determined by that proposed, that of measuring the external forces separately. Many illustrations of this dynamic action might be offered; they will occur to the mind of every sailor who reads this. One, however, may be given.

Captain Hoseason mentions the case of a merchant-vessel that was assisted by a ship in which he served. The former, off Cape Horn, was struck by a sea, which carried away her poop and its contents, breaking off every timber-head but one.

The captain, his wife, and the officers, were sleeping there, all of whom were swept off and drowned, except the captain's wife, who saved herself by catching hold of the solitary timber-head.

This ship probably had her centre of gravity too low, therefore was too unyielding, that is, would not roll with the sea.

Had she had only the amount of stability designed by the Froude-Reed system, the sea would have capsised her, when she would have become a total loss!

Arising from the causes above stated, we are without any actual measures of stabilities, nor can we directly compare the stabilities, nor the performances of one ship with those of another by the existing system.

It may be asked, how is it then that we have gone on so long without comparative failures till Mr. E. J. Reed became constructor? The answer is simple and complete.

The amount of stability provided for in the estimates of designs was so large that it admitted a margin for errors to occur without their being discovered, and the estimates were all understated, because the concentration of weight on the bottoms formerly, and the comparative lightness near the water-line, occasioned greater actual stability than the calculations indicated.

But as Mr. Reed reduced the initial stability, as shown by the old rules, and studiously concentrated weight on the sides and near the water-line, and more studiously raised the weights off the bottoms, making them light, on the plea of limiting the rolling and for economy of material; his designs are numerous, and many of them conspicuous failures as to stability.

We have now the "Raleigh," of 4,780 tons, 6,000 horse-power, and its consequent great weight of coals, drawing 16 inches more water than was intended, and receiving 180 tons of ballast, to make her useful, if not to make her safe.

True, it is stated that her weight of armament had been increased, but this is not said to have been the cause for introducing the ballast, nor could it with truth be so, as the former weight could not have been great, and some of it added much below the centre of gravity. Fifty tons would cover the increase of top weight.

If we reduced the old sailing three-decked "Trafalgar" to a frigate, which the "Raleigh" is, we should remove her poop, upper and main decks, with their topsides and armaments, the latter equalling that of the "Raleigh," we should reduce the top weight of the former from 700 to 800 tons, yet she carried this great weight well, possessed great stability under sail, was easy, and rolled by much the least of any ship in the squadron. Marvellous to relate, the "Raleigh," of greater dimensions having a tonnage of 4,780, as compared to 2,700 of "Trafalgar;" notwithstanding, also, the great weight, 800 horse-power engines and boilers, and

550 tons of coals, being comparatively low down; and, notwithstanding the lowness of her small armament, say 150 tons, as compared with 330 tons of "Trafalgar," she requires 180 tons of ballast between her bottoms!

The explanation is, that the no-stability mania led to her being deprived of margin enough of stability to cover the errors of calculation, and her weights are lifted off her bottom, leaving a buoyant capsizing space below.

In creditable contrast, we observe, that an iron sailing ship, built for Messrs. Stuart and Douglas, of Liverpool, by McMillan and Son, Dumbarton, of 4,500 tons, was launched with her top-gallant yards across, without a particle of ballast in her, and manifesting considerable stability during a gale which occurred immediately after she was launched.

Taking into consideration the greater weight of this ship's bottom plates, and much greater strength and weight of the lower part of her frames, the centre of gravity of her hull must be very low, and with a reasonable stowage of her cargo, it must approach near her centre of gravity of displacement.

It is therefore not within the bounds of probability that she could be made unsafe by the stowage of the weight of her cargo too high, nor would a deck load, more than the double of "Raliegh's" armament, have made her unsafe, or have necessitated any, much less 180 tons of ballast. Her length being six times her breadth, she will be an easy and useful ship, and will condemn the no-stability upside down system.

The fact is, that if so much fuss is made of an increase of top-weight of only 50 or 100 tons to ships of 4,700 tons displacement, the Navy will be the laughing-stock of the mercantile marine; and if such large ships cannot bear so small an increase of top-weight without requiring 180 tons of ballast to restore the stability thus lost, it shows how dangerous was the "Raleigh" as designed, and the wisdom of keeping her months in leading-strings while they are teaching her to walk safely.

Can it be just that merchants should be criminally prosecuted if their ships capsize and drown their crews, because of a deck load placed on a crank ship, when they are less culpable than the architects of the Admiralty, who have built ships that were liable to capsize without a deck load; surely this involves grave responsibility.

We rejoice that ballast is being placed in these ships, but surely public notice ought to be given of this, that the naval architects of the mercantile marine should be informed as to the dangerous character of that system which previously prevailed, and had been inculcated as the only safe system!

It is quite clear that till there is a new and correct system of calculating stabilities and rolling motions, there can be no guarantee for certainty of results, for progress, for efficiency, economy, or safety.

When single ships cost 5,000*l.* or 50,000*l.* the failures were few, and did not involve much expense or loss of life; but now, that we have ships costing nearly 300,000*l.*, few in number, consuming collieries, and with large crews, it becomes a great national question. Horrible as has been the responsibility of those who originated those failures, which, for number, design, and grandeur, are without parallel, it will be nothing to that of those who, after such sad experience, should continue the looseness of practice that we described, or should repeat the failures which have of late been the rule in the navy, the result of that revolution in naval architecture which Mr. E. J. Reed boasts to have effected, and from the grievous consequences of which nothing short of another revolution can save us.

The "Stuart Hahnemann," to which we alluded above, has a metacentric height now of about six feet, and if her cargo be stowed, as all ought to be, if mixed, and not a cargo of metals, the heaviest portion lowest, her centre of gravity will be, when she is upright, at the same height nearly as that of her centre of gravity of displacement, the position recommended, when it is possible, by that eminent naval architect, Dupuy de Lome—"*Son centre de gravité à peine au dessous de son centre de carène.*"

Let such an amount of stability be given, and ships will be safe from capsizing, will sail and steer well, and then, in comparison, in respect of safety, the lowness of freeboard will be as nothing.

No doubt the water may wash over the decks, but if provision be made, as in the Dutch galliots, by great stability and high buoyant ends, the water may wash over a length of deck of 20 or 30 feet amidships without danger.

In the Froude-Reed system it is actually intended that the sea shall come over the bulwarks, in the arrangements to keep ships upright in a seaway, but to the chances of being swamped if the hatches are not securely battened down, and this cannot be done without suffocating the crews, who sleep in the high ends of the galliots, is added in the Froude-Reed designs, the next to certainty of being capsized from their having so little stability.

The Scientific Committee seem to have had some fears of the former danger, for they expressly say, care must be taken that the "Devastation" should not be "swamped."

If the Board of Trade and Admiralty were to inculcate the principle of giving great stability, instead of the little stability recommended by Messrs. Froude, Read, and some of the Scientific Committee on naval designs, they would do more to limit the number of ships that founder and drown their crews than all the Acts of Parliament that could be framed as to deck loads, load line, or height of freeboard.

The "Raleigh" is the last, in one sense; we hope she may be so in in a more important sense, of the unsafe fleet of ships.

It had been supposed that the great weight of engines, boilers, and coals, low down, would have sufficed for ballasting those ships that carried such—not so. "High science" designs require from 180 to 500 tons of ballast still lower down, not to make them stable and safe,—that is a vulgar error,—but for an economical adjustment of weight!

It is proposed by the self-styled school of great thinkers to place one of the greatest authorities in the world on the subject of Naval Architecture in uncontrolled authority over the Constructive Department. The grounds for the recommendation we are not told, whether to correct the failures in the designs to prevent ships rolling, the effects of which as respects the stomach, Mr. Bessemer is called in to cure by machinery, or to carry forward the anticipated triumphs of the so-called high science?

We had thought that any, who could be convinced by reason and hard facts, would have been satisfied that ballast and ballast had sufficiently witnessed to the danger of placing power in hands that have violated alike common sense, experience, and laws.

Surely foreigners will say we English are a strange people; we appoint grave Commissioners to conduct competitive examinations for offices the duties of which might be creditably performed by army pensioners, and yet we hoist men into the highly responsible position of constructors and advisers to the Admiralty—men without experience, without this competitive examination, nay, even without any examination; men who, if they have been previously tried, it has been far other than a success, and old public servants, of proved competency, resign rather than be responsible for the evils they anticipate from such. The results have fully established the right of these faithful servants to honourable mention, if not also to a more substantial recognition of their conduct.

Till the law is made to reach over the high seas and back to the designers of fantastic ships, sailors' lives will not count for much.

INTRODUCTION.

It was not prematurely that we undertook to examine the current opinions on Naval Architecture,* when we found not simply a recommendation to *reduce* the *stability* of ships with a view to *make them more stable*, but this wild scheme extensively entered upon in opposition to solemn warnings from experienced naval architects and mathematicians skilled in practical naval matters.

Thus Mr. Scott Russell said, in 1863:—"Mr. Froude "has recommended that ships should be constructed so as to "have the largest possible periodic time of roll, and has "recommended as the method of giving this long periodic "time, the lessening of her stability under canvas. I have "carefully examined the subject with reference to the safety "of following out such a principle, and I have compared it "with the results of a long course of practice of my own, and "have come to the conclusion, that both in principle and in "practice it would be *unwise* and *unsafe* to follow his advice."

Mr. Froude recommends, for "insuring the safety of a "ship, as a practical measure, that it should have given to "it such a distribution of weight as shall insure to it a long "period of oscillation, and he adheres to this maxim under "conditions and to an extent which to me appear *dangerous* "and *unsound*. What I assert is, that such a cure is worse "than the disease.

"I do not think that this synchronism of oscillation "(which Mr. Froude fears) is a formidable fact, or is an "ordinary source of danger to real ships on real sea waves, "as distinguished from experimental models in a fish-pond."

The Rev. Dr. Woolley said at the same period, "I may "safely say that in the main I agree with Mr. Scott Russell's "remarks.

* *Vide* Lecture on the loss of the "Captain." Current Fallacies in Naval Architecture. Our Ironclads.—E. and F. N. Spon, 48, Charing Cross.

"Those who were present at the discussion of the paper "by Mr. Scott Russell, cannot fail to have been struck by "the wisdom of the concluding remarks of the Chairman, "Canon Mozeley, in which, while allowing fully the merit "of the investigations of Mr. Froude, he bade naval archi-"tects be cautious in applying his conclusions, which are "sufficiently in antagonism with all that have been hereto-"fore formed with regard to the form which conduces to the "best qualities in a sea-going ship. In these words of cau-"tion I fully concur, and for this reason, that Mr. Froude, "in pressing the conclusions of his own theory, which is "entitled to all respect, seems to forget that, after all, his "mode of viewing the question is but one among several."

The lamentable fate of one, and the facts as to many, of our ironclads, have fully justified these warnings.

The concentration of weight on the sides of these ships also caused danger by further depletion of stability than was anticipated: and more dangerous still, the calculations made were based on a hypothesis which we have shewn frequently, but now more fully, to be either inapplicable or altogether erroneous.

These calculations proceeded, 1st, on the hypothesis that the weights in the immersed portion of the body were uniformly distributed, which never was the case, but, as formerly, the excess of weight was on or near the bottoms of ships, the error was on the *safe* side—increasing the stability *above* the amount assigned by the calculations thus made.

In consequence of Mr. Froude's propositions, *deep* empty spaces in the bottoms of ships were introduced by Mr. Reed, as he stated in his Lecture at the Royal Institution, "expressly to facilitate the raising the engines, boilers, and "other weights, because it has been ascertained that the "tendency of ships to roll has been reduced by these means;" this caused the errors to be on the *unsafe* side, that of decreasing the stability *below* the quantity assigned by the calculations. 2nd. The calculations proceeded on the further hypothesis that the centre of pressure of the water is always at the centre of the immersed figure, whereas it is demonstrated, in Appendix A, that it is always much lower;—the effect of the former is to assign to every ship greater stability than she possesses.

With the distribution of weight formerly practised and the then mode of calculation, there were two errors; one counteracting the effect of the other, and thus giving a result near the truth, and so the old ships were comparatively safe, though the reason was unknown, as the moments of the sail agreed well enough with the moment of stability thus deduced, it was supposed that the method of calculation by which the latter was arrived at was correct.

But when the weights came to be distributed on to the sides and empty bottoms given to ships, both errors were brought on the unsafe side, that of assigning, for two reasons, a greater stability than they possessed.

These were the reasons why the "Sultan" and "Invincible" class required so much ballast to make them less dangerous, and not, as was stated, because an equal weight had been omitted from their hulls in the course of construction.

This is an untenable afterthought, for Mr. Reed, in his letter in the *Times* of November 7, writing of the "Vanguard" class, wrote, "it would probably be necessary "to correct the centre of gravity by ballast, observing that "100 tons of cheap pig iron in that form would serve the "same purpose as 200 tons of expensive iron distributed "throughout the hull, and wrought into the structure."

Therefore, on this premiss, 360 tons of ballast, the quantity placed in these ships, would require the omission of 720 tons of iron from the hull generally, during the construction, to justify the assertion that this amount of ballast only placed the centre of gravity where it was intended to be—yet, no one pretends that even half this quantity was omitted, and it is incredible that 500 tons could have been omitted from the bottom of the "Sultan," so low down as where that quantity of ballast was placed, or 360 tons from the bottoms of the "Invincible" class, still more impossible that these quantities could have been omitted consistently, with an adherence to the specifications under which these ships were built, or with the sanction of the Admiralty Officers superintending their construction.

Moreover, we have an *entirely different explanation* given in the Blue Book, which states that "the metacentric height "of the 'Iron Duke' was three inches higher than that of "the 'Vanguard,' and that there was a difference of five "inches in their immersion," adding, that "this difference "in the metacentric heights of the two ships may be due "to a difference between the actual and the calculated "weight of water ballast in the 'Invincible,' or to a small "variation in the dimensions and form of the ships, or in "height of decks, or to alterations in the detailed arrange-"ments of works of hull, or from these causes combined, all "of which often operate to cause some dissimilarity in ships "built from the same designs."

The fact is, no reasonable explanation has yet been given by these gentlemen of their great blunder. That such it was, is clear from what the First Lord of the Admiralty said in the House—"an error was made in the construction of those ships, and to correct that error it was necessary they should carry 300 tons of ballast."

How unjustifiable was the reduction of the stability below that of known good ships, and particularly so low as that nominally represented by two feet of metacentric height, may be gathered from a report of the late Controller, who, after comparing the rolling of the ships of two squadrons, English and French, said the "'Solferino' stands "alone as a type of excellence," and the same report gives her metacentric height as 4·5 feet:—more than double the height assigned to the "Vanguard!" And when allowance is made for the fact that the "Solferino's" weights were down on her *heavy* bottom, her true metacentric height must have been quite four times greater than that of the "Vanguard" or "Sultan" class, and yet we have been building very low freeboard ships,—with only 3·5 feet of metacentric height, and even less, and these with light bottoms and empty spaces.

We are told in *Naval Science* that 8·64 feet, the metacentric height of the Russian ship, "Peter the Great," is nearly double that of the "Devastation," *i.e.*, at least 4·32 feet. The *Times* correspondent informs us, on authority not to be disputed, that the "Devastation's" metacentric height is 3·5. We cannot believe that the addition of topsides and the increase of accommodation for the men could have occasioned this difference, for though she has a "deck load," yet 400 tons of coals have been omitted.

If this top weight has had the effect of reducing her initial stability by the large amount these figures imply, it is an instructive comment upon the arguments to prove *her* more safe for the operation she has undergone; this is surely heroic treatment, kill or cure.

It is said that on one occasion the "Devastation" gave so deep a lurch that all hands rushed on deck thinking she was not going to rise again; possibly had she been struck with a sea at this critical moment, as the "Captain" was, we should have had another proof of the folly, to use a mild term, of giving ships small initial stability and deep empty spaces in their bottoms.

We by no means intended by what we have written to imply that no ships are lost because of being badly constructed or from being ineffectively repaired, or from being insufficiently found, or that there was no need for the inquiries initiated by Mr. Plimsoll. We think there is great need for interference and for a limited amount of legislation, but we are fully assured that the owners and underwriters are not always, or the only people, responsible for the preventible disasters amongst our shipping.

We have shewn, page 75, Appendix A, the enormous effect of a mistake to the extent of one-eighth of an inch in the length of the lever of stability. Nor can it be otherwise than that there will be great and fatal mistakes, for the means employed to ascertain the length of this lever are erroneous.

This lever commences and increases with the angle of inclination. When a vessel is upright, the centre of buoyant support is in a vertical under the centre of gravity, and as the vessel inclines, one side is more immersed, the other side emerged more, consequently the centre of buoyant support passes over to the side that is most immersed, and then the line of support passes up on that side of the centre of gravity—the horizontal distance of the centre of gravity from this line is the lever in question.

The length of this lever depends on several conditions; the amount of inclination, on the form and volume of the solids of immersion and emersion at any given angle, and upon their specific gravities, together with the differences between their specific gravities where they differ, and the specific gravities of the remaining portion of the body.

Now the calculations which we affirm to be dangerous proceed on the assumption that the solids above alluded to are equal isosceles and right-angled triangles, that a ship rotates round a fixed point in the middle of the load water-line, and that the new water-line bisects the old; that there is no rise or fall of the centre of gravity as the ship inclines, and, with other assumptions, that the multiplicity of minute measurements made on a very reduced scale are accurate.

Yet these are never true.

Moreover, these calculations proceed on the further assumption that the specific gravities of these solids are alike uniform with those of the remaining portion of the body. This assumption is founded on another, *i.e.*, that the immersed body assumes the conditions of the fluid it had displaced. We may ask, if so, why has it not assumed its fluidity?

And then proceed on the further assumption, that the pressures on the immersed portion of ships are always uniform. This is not so, for as the ship assumes the *place* of the displaced water, she is subject to pressures increasing with the depth.

We need hardly say that these unreasonable assumptions are all contrary to fact, and are more extensively so in iron-clads with heavy armour and empty bottoms. Therefore all calculations as to the length of this lever thus made are erroneous, and, because of assigning a much greater length than is the fact in ironclads, are extremely dangerous. We have seen in proof of this, that there was a difference in the metacentric heights of the sister-ships "Iron Duke" and "Vanguard," amounting to 3 inches, or one-eighth of the whole quantity. This represents a great many one-eighths of an inch. To account for this, a number of things were suggested, one or more of which were supposed to have been the cause. It might have been added with truth, that none of them may have contributed to the result. We are not told why the omission of weight from the bottom during construction was not suggested, as in other cases, as being the cause.

We have seen also the enormous blunder in all these ships, requiring 360 tons of ballast to correct. With equal propriety it might have been suggested that, as in the "Iron Duke," increased dimensions may have been the cause of the reduced immersion of the "Vanguard" and sisters, instead of the convenient assertion that it arose from the omission of an impossible weight from their "very bottoms."

All this inconsistency and looseness establishes the unreliableness of the current modes of calculation. Moreover it is said that in action the ends of the "Inflexible," which are unprotected by armour, are to be filled with water, and that "directly the water is allowed free access to the ends "above the armoured deck, the only reserve of buoyancy is "that afforded by the central citadel, but this will be assisted "by a belt of cork, 9 or 10 feet in depth, resting on the "armoured deck—a plan suggested by Sir Wm. Thomson."

That is, when by this increased immersion the ship is in greater danger of capsizing from being deficient in stability or from being run over by a ram, she is furnished with this cork to save her! The attempt is certainly due to her crew.

This *life*-belt will be efficacious in proportion to its volume, nearness to the water-line, and its horizontal distance from the centre of gravity, together with the smallness of its own specific gravity.

The principle is that which the writer has been years contending for, and whether it be perceived or not, it is clearly an admission as to the correctness of views herein and long since enunciated by him, and likewise an admission as to the *incorrectness* of the received mode of calculating stabilities.

We are assured that the only hope of safety from numberless disasters, both in the Royal Navy and in the Merchant Service, lies in the adoption of the methods we propose in lieu of the dangerous assumptions we have exposed.

It may be asked, how it is that the system objected to has served for so many centuries? The answer is easy and complete. Owing to the heavy bottoms and to the heavy weights being placed down in them, while buoyancy was preserved near the load water-line, there was a large margin of error on the *safe* side which was not seriously affected by smaller errors that occurred on the *unsafe* side.

When, in accord with the Froude-Reed destitute of stability system, ships were built upside-down, *i.e.*, with light bottoms and empty spaces in them, and with an accumulation of weight near the load water-line, the margin of error was moved to the *unsafe* side, and though these ships were only saved from capsizing by a reversal of the principle upon which they were designed, their designers will not be persuaded that their fatal disease was want of stability, but still contend that their scheme was good, because these ships possessed, equally with the French "Océan" class, a nominal stability represented by 2 feet of metacentric height, and the French ships were sailed and did not capsize, forgetting that their metacentric height was 2 feet margin on the safe side, = 2 = 2, or 3 feet, while that of the "Vanguard" class was 2 feet − the margin on the unsafe side = nothing or little!

The Admiralty Constructors say truly, that "their calcu- "lations do not enable them to determine what is the amount "of *stability* which is necessary for a ship to render her *safe* "as a sea-going sailing ship. We can merely compare the "amount in one ship with another."

We have demonstrated that they cannot do even this latter, that their comparative estimates are radically wrong and dangerous; moreover, that the assumptions upon which their calculations are based, *i.e.*, that a ship revolves round a point in the midship section at the water-line or round the centre of gravity, and that an equilibrating lever, or lever of stability, giving a correct estimate of the stability, is attainable by their system, are untrue and as little worthy of belief as is any other of the most untrue of popular superstitions.

For even were their premises true, they are not worked out with mathematical accuracy. But their premises are not true, inasmuch as they assume an axis and a point in this axis as the origin, a point which has never been determined, and that which they have assumed is not the true one.

Therefore, to continue to design ships on such an hypothesis, and to support colleges and schools to extend and give permanency to such views, is to hinder progress and to guarantee waste of life and treasure.

Meanwhile we have shewn much as to the chaotic condition into which naval construction has been brought, naval architects and mathematicians practised in naval matters being our witnesses, and while we have demonstrated the great danger of specific errors arising both to the Navy and to Merchant Ships, we have indicated the direction in which a remedy is to be sought.

We have placed the purely mathematical arguments in an Appendix, giving those only in the body of the work that may be easily comprehended, as we contend that no officer, whether of the Navy or of the Merchant Service, should be without a knowledge of the subject.

We have selected the simplest forms for illustration, and all of exactly the same dimensions and form, and have arranged the weights in all the figures so that the centre of gravity shall be in the middle of the load water-line of each, that thus the question should be divested of complication from there being only one variable.

We have given diagrams of a typical ship formed by definite mathematical curves, horizontal and vertical, for the purpose at once of calculating all their elements with accuracy and affording greater facility for making them. The curves may be adapted to any dimensions, and should it be found necessary or be desired to adopt a different form, this can be done without difficulty.

The centres of pressure are given for each quarter; from these can be determined those centres for the system combined.

The computations are made on the hypothesis that the weights in the immersed body are equally distributed. Any departure from this condition should be allowed for.

Any departure from the typical form should be allowed for.

The calculations give absolute, instead of mere relative, quantities as by the current system.

Unreliable calculations, that now occupy two months to complete (see Evidence, "Captain's" Court-martial), may be made with accuracy in two days.

Owing to the fact that hundreds of men are yearly being drowned in consequence of the foundering of their ships, and the Plimsoll Commission being unable to recommend any legislative measure to protect them, doubtless from the conflict of evidence as to the causes of these sad losses, we felt that it was imperative at once to publish, leaving for a subsequent publication more extended proofs and illustrations of the truth and value of the system herein enunciated.

We have thus far done that which we believed to be imperative for the protection of life and the benefit of our country: nevertheless, we shall not be surprised to find men repeating their endeavours to mislead the public; knowing, however, from communication with scientific men in this and other countries, that we stand not alone in our hostility to the dangerous conceits introduced of late into Naval Architecture, we shall be unmoved by the vilification of those who are unable to answer our arguments and who cannot see that it is for their interest to accept them—resting assured that our views must eventually prevail.

DISCUSSION.

1. THE all-important question is, are our ships safe? Do the facts justify the conclusion that they are so? We think not. And if we continue under a false security, misrepresentations and evasions can only make matters worse.

2. That there exists a wide-spread and well-grounded distrust is clear, for we have—

3. A 9,000-ton ship sent to sea, with a small ship as a nurse to her, for so small and so different a vessel could not have been intended as a standard of comparison.

4. A number of great ships only are saved from capsizing by being very heavily ballasted.

5. An officer, many years in the Constructive Department of the Admiralty, says, "the loss of the 'Captain' caused "very great attention to be paid to the curve of stability: we "never had given much attention to it before;"—than which a more damaging admission to the reputation of a great building department could not be made. The loss of a huge ship of war, with 500 souls, necessary to stimulate the responsible authorities to give much attention to the vital element of stability!

6. Then we have J. C., an able writer, saying in reference to a paper by Professor Rankine, entitled, "Remarks on the "Stability of Mastless Ships of low freeboard as affected by "the Waves," based upon a formula obtained by Mr. Froude, framed on the following assumptions:—

"1. That waves are all equal and of a uniform period.

"2. That the waves are all of a definite trochoidal form.

"3. That the reaction of the water-pressure on the ship "is always perpendicular to the surface of the wave at the "part occupied by the ship.

"4. That the time of oscillation of the ship in smooth "water is the same for all angles."

J. C. says, truly, none of these assumptions can be regarded as accurately true, and one or two of them are plainly very rough approximations to the truth. For this reason I have always regarded Mr. Froude's formula as exhibiting the general aspect only of the rolling of ships in waves, and not as affording any trustworthy means of calculating the extent of the roll in any practicable case, and adds:—

"At present I do not see a way to overcome these objec-"tions and difficulties, nor should I regard either method "(alluding to one of his own) of investigation as affording "trustworthy means of calculating the precise amount of "stability which would be required in a new design. The "only safe guide in this matter is, in my opinion, found in "experience with successful ships, and in designing new "vessels, it appears desirable to provide that amount and "range of stability which have proved sufficient in ships "that have been thoroughly tried."

7. Further facts and proofs might be given, but suffice to say that opinions have been adopted which revolutionized naval designing: that element in ships which had hitherto been considered indispensable, vital, came to be thought an evil, and to be given in a dangerously insufficient degree; and yet before what we would call such an incredible theory was acted on, records should have been produced of many ships having been capsized from their having possessed too much *stability*, yet not even one is produced from the whole range of history!

8. J. C. observes, "The waves of the sea present phe-"nomena as abnormal as those which gusts and squalls "present in air, these phenomena occurring chiefly in the "form of great exceptional sea-gulfs (negative waves), "followed by no less exceptional sea-hills (positive waves) "the two together accounting, we think, for many of those

"disasters at sea which common waves, so far as we know, "do not account for."*

9. Therefore, a return to the old and safe custom of giving a considerable amount of initial stability cannot be too much or too soon insisted on, as we shall now proceed to prove. Mr. Reed—for we must hold him responsible for the anonymous publications in his magazine—states that "the "'Invincible' class possessed a metacentric height of about "two feet before they were ballasted, and that with ballast it "was three feet: that it was considered desirable to add "ballast in order to give greater initial stability, *not* to afford "increased safety."

10. The Admiralty authorities state that the metacentric height was somewhat less in each case. This metacentric height is supposed to give a just estimate of the stability.

11. Few acquainted with the facts would admit the correctness of Mr. Reed's view, or would admit the reliableness of the calculations on which either opinion was founded.

12. There can be no doubt that while unballasted they were officially declared to be "unseaworthy," and though they were so largely ballasted, by 360 tons, the authorities deemed it necessary to reduce their yards and area of sail, notwithstanding that these were originally only of the dimensions given to sailing ships of not more than half the size of the "Invincible."

13. Before being ballasted the "Invincible" was reported to have heeled 10° in the operation of turning, and 17° when this operation was repeated with the addition of a moderate wind on her side, without a stitch of canvas set, or all of her upper deck weights being in place. We have only to imagine sail placed on her, followed by a sudden gust of wind, which often doubles the inclination, together with only such a sea as was running when the "Captain" was lost, in order to present to our imaginations the inevitable fate of these ships under such circumstances.

14. This frightful danger is incurred in order to guard

* No doubt many disasters that exceptional seas are made answerable for arise from insufficient stability, the result of calulations based on erroneous data.

against an alleged remote danger, that of being rolled over by an accumulation of inclination gained in successive rolls, and to obtain ease of motion for working the guns.

15. Surely to escape a problematical and distant danger does not justify the creation of an immediate and constant danger, with many other disadvantages in sailing and steering, and one is at a loss to conceive how an inclination of even less than 10° could do other than reduce the facility of working guns, while with so little stability the mere running out of her guns must have inclined her so as to make firing to any useful purpose well nigh impossible; while the guns being run in and out could not fail to roll these ships so much as to destroy all accuracy of fire. Then, with such limited amount of stability, these ships could neither steer nor sail well.

16. It is clear, from the extensive alterations made in these ships by the Admiralty authorities, by the reduction of their sails, and by giving them such a very large amount of ballast, that *they* did not consider them safe with a stability represented by nearly two feet of metacentric height; surely, then, it is more than doubtful whether less than two feet can be a safe amount for other ships!

17. The "Monarch" is shewn to possess a metacentric height when her coals and provisions are expended of only 1·28 feet, and only 2·43 when loaded. How can this be a safe quantity, when ships with 2 feet are found to be unseaworthy, if the current metacentric method gives a true measure of stability? No doubt a small reduction has been made in the area of the sails of the "Monarch," but this is clearly not enough, for this reduction does not affect her condition in bad weather, when the danger is greatest.*

18. Then we have the "Inconstant" with 2·2 feet of metacentric height when loaded, and 1·23 feet when light, but with 90 tons of ballast in. Can she be safe? 90 tons more,

* Even a small amount of ballast low down in the "Monarch" would make her vastly more safe, and would improve every one of her qualities; this would have been the course to have adopted instead of that of reducing her area of sail.

it is said, has been given to her, all the better, but she is reported to heel 15° under plain sail; if this be so, we have but to imagine her caught by a sea on the weather beam, when thus inclining, with a summer squall also, and we shall have imagined more than enough of probable causes to capsize her; and yet we believe that she, even with her greater area of sail and smaller size, is safer than the "Monarch." How very much worse would be the condition of either if caught in a hurricane?

19. True, *Naval Science* informs us that the "Océan" class, the French rivals of the "Invincible," have a metacentric value almost identical with that which the "Invincible" class had before they were ballasted, and the "Océan" class are reported to be capital sea-boats, "steady, and well-behaved, "but wanting in stiffness." So far this is in favour of my view. He adds, "They are not in as good case as the "'Invincible' class, for the draught of the French vessels (said "to be 29 feet)* has so exceeded that designed for them that "increase of stability by ballast is impossible."

20. But the fact is, the conditions of the two classes are so radically different, that no just comparison by the metacentric method can be made between them.

21. The "Océan" class are wooden ships, having heavy bottoms, and much of their weight down on them, with low centres of gravity. The "Invincible" class are iron ships with light empty spaces in their bottoms, and a high centre of gravity. The former would have a centre of buoyancy below the centre of her immersed body; the latter would have a centre of buoyancy above the centre of immersed figure, therefore, the actual stability of the former would be greater than that of the latter, yet the metacentric method assumes that the centre of buoyancy of each was at their respective centres of figure, and is, therefore, erroneous. In fact, the condition of the former is more analogous to that of the latter *after* they were ballasted than before, as I shall subsequently prove.

* This great draught explains her easiness without supposing her to have a high centre of gravity, which no doubt is not the fact.

22. But to obtain a more full estimate of the danger of our ships, we have but to imagine them caught in a hurricane, such as that which threw the "Conqueror," a fine 90-gun ship, on her beam ends, though she had not a stitch of canvas set at the time.

23. She was only saved from foundering because she could float long enough to be swung round, when the wind taking her on the other side set her up on her legs again.

24. And yet her metacentric height was not simply 2 feet nor 3 feet, but 4·6 feet!! And who is prepared to say that a less force of wind and sea, would not have thrown her as effectually over, and who will venture to say that any ironclad would have survived the crisis the "Conqueror" passed through, and yet 4·6 feet does not give the full measure of her stability as compared with that of ironclads of the same measure, estimated by the metacentric method in use, for reasons which I will subsequently point out. No doubt some will think that there is assured safety in smaller masts, or no masts, and that, therefore, equal metacentric height may be dispensed with; to these I would say, the second time the "Racer" was capsized she was without masts, having lost them on the occasion of her first capsize. No doubt it will be found she had little or no ballast, water, or provisions, and arising from her form her centre of gravity must have been high, high enough to please even the most rabid advocates of high centres of gravity.

25. Thus, supposing the mode in use of estimating stabilities to be correct, even then, these ships are all *unsafe* against such contingencies as those to which they may be subjected,—not frequently, perhaps, but still sufficiently often to imperatively demand security against them.

26. While there are such contradictions and anomalies arising from what, at best, is but a system of trial and error, without rule; it is incumbent upon all to urge an examination as to whether the metacentric height, as determined by the current method, affords a correct measure of stability.

27. The evidence goes far to prove that it does not; and that neither the "Invincible" class as designed, nor

"Captain," nor "Monarch," nor indeed any of our iron-clads possess the amount of stability assigned them by their respective metacentric heights.

28. The "Captain" heeled 4 degrees by merely turning her guns round to the leeside.

29. The first broadside fired by the "Captain" made her roll 2° each way.

30. The "Monarch" "cannot lay her guns fairly when all "are run out on one side, owing to the heel they give "her."

31. Now, it is impossible that such small weights moved such small distances could heel vessels of 6,000 or 7,000 tons if they possessed the stabilities that 2·6 feet or 2·43 feet of metacentric height is said to indicate.

32. Mr. Pearce, of the firm of Elder and Company, "always found 'Invincible' sit upright, but, as she went "into Plymouth before ballast was put in, sitting on her "bilge. Supposes she has *three* positions of stable equili-"brium; believes she would have proved stable if unmasted."

33. How is it possible she could have sat on her bilge, or have had three positions of stable equilibrium, if she really had possessed the amount of stability indicated by nearly two feet of metacentric height? or how could she have manifested the extreme tenderness or tendency to heel easily, reported by other witnesses, if she had had the determined amount of stability represented by two feet?

34. Yet the "Invincible" class was designed, not only to be masted, but also to carry a large spread of canvas.

35. There are other and accurate methods of estimating the amount of actual stability; between these and the meta-centric method we find enormous discrepancies.

36. Thus when the "Sultan" was first tried at the measured mile, she heeled, in turning, 10°, and at her second trial, after nearly 500 tons of ballast had been placed between her bottoms, she heeled only $1\frac{1}{2}$°. That is, an equal and similar force occasioned more than six times the amount of inclina-tion under the former conditions than it did under the latter. In other words, her stability, allowing for errors of observa-

tion, was six times as much under the former, as under the latter circumstances.

37. We do not know what her metacentric height was in either case, but it cannot have been very different from that of the "Invincible," relatively it may be considered alike.

38. The "Invincible" inclined 10° also, in turning, and, after 360 tons of ballast were placed between her bottoms, also in turning, she heeled only a degree or so. That is, her stability in the one case was more than six times greater than in the other; yet we are told that her metacentric height when she heeled 10° was nearly as much as 2 feet, and that when she heeled very little it was only 3 feet. In other words, the metacentric height being the measure, her stability was said to be increased by the ballast only in the proportion of 2 to 3 instead of as 1 to 6, as given by the angles of inclination when turning.

39. It may be said, that the conditions that each ship was subjected to on each occasion of turning were not strictly similar; it may have been so; but this is an argument that will cut either way, to increase the difference or reduce it. The error could not have been great, and would have been only one of degree, and not of principle.

40. It has been said that the centrifugal force, which does not obtain when the ships are inclined in the basin, will account for the great differences in the inclinations recorded, and the results as to stability given by the two systems, but it is difficult to imagine any difference on this account, certainly to suppose any of considerable amount, since the centrifugal force obtains in both cases of turning, and it is greater in proportion to the quickness in turning, and we think it will be found that vessels will turn quicker when the angle of inclination is least; and, if this be so, the increase of stability produced by the ballast is greater than the differences of angle of inclination indicates.

41. The centrifugal force in turning takes the place of the inclining weight used in the basin.

42. Whatever may be the inclining force or forces, a sufficiency of stability must be given to resist them, or danger

will ensue; and, taking the estimates given by measuring, the effect of the centrifugal force is the safest as truest; for these are observed facts that multitudes can verify, while the estimates from the experiments in the basin are but opinions founded upon calculations made on an erroneous hypothesis.

43. The force of the argument proving the incorrectness of the current metacentric method, as conducted, is greatly increased by the fact that the actual total stability of a ship is greater when she is in notion than when it is measured in the basin; for, in the former case, there is the same amount of statical stability, but to this is added a hydrodynamic stability proportional to the speed with which the ship is moving.

44. We have seen a record in the *Times* of the manner in which the squadron of ironclads tumbled about, tearing themselves to pieces, for want of this headway and addition to their practical stability, immediately the amount of disturbance of the sea was sufficient to overcome the inertia of their iron sides; proof was thus given that they did not possess the amount of statical stability that the metacentric method had assigned them.

45. It is an every-day occurrence, also, to see a boat that has lost her way from losing the wind through passing under the lee of a ship, nearly capsizing as she catches the breeze after she has passed to the other side of the ship, but standing up under her sail more and more as she gathers way again.

46. Nor can it be said that the estimate given by the basin-inclining method, as practised, is on the safe side, because a vessel, when in motion ahead, possesses a greater total stability than her inclination indicates, for it is necessary to provide for sufficient stability when a ship is without motion ahead, and, therefore, without hydrodynamic stability.

47. We must not be understood as objecting for the present to the use of a metacentric method, but as holding that the present method gives too high an estimate of the stability of all ironclads, and especially of those that have deep empty spaces at or between their bottoms, and it is

inapplicable to them, as has been proved by the great inclination of such ships in turning, wherein such a great deficiency was shown, notwithstanding the great addition made to their total stability by their great speed. The present metacentric method, therefore, requires to be modified, so as to obtain a true quantitative measure of the statical stability.

48. To the question, Is not the present system safe? we may answer by another, Can there be safety where there is ignorance? That there is ignorance, the present Board of Admiralty Constructors testify, when they say, "We "have already pointed out that our calculations do not enable "us to determine what is the minimum amount of stability "which is necessary for a ship to render her safe as a sea-"going sailing ship. We can merely compare the amount "in one ship with another." And, it may be added, this cannot be done, even approximately, when the ships are radically different.

49. The clear practical fact is this, that, when without ballast, these ships inclined to a dangerous degree, and that whatever may have been the inclining force it matters not. The previous calculations made concerning them were utterly at fault, neither did the calculations made on their inclination in the basin give any sufficient estimate of their danger or of the true value of their stabilities, so that probably had it not been for the previous loss of the "Captain," one or more of them would have been sent to sea as designed, and have also capsized and drowned the crews.

50. It is impossible to believe that the metacentric method as conducted can give a true estimate of stability, since it assigned a metacentric height of two feet equally to the "Invincible" class and to the French "Océan," and to the "Inconstant;" while the "Invincible" inclined 17° in turning with a moderate breeze and without a stitch of canvas set, the "Océan" class are able to carry all sail safely and win the character of being capital sea-boats and well-behaved. And the "Inconstant," a full-rigged ship, carries all plain sail, which is one-third greater than that now carried

by any of that class, and without heeling to so great a degree as the "Invincible" did.

51. Taking 3 feet as about the metacentric height of this class since they have been ballasted, that before they were ballasted could have been only one-sixth that height, or five-tenths of a foot; yet even so much is hardly compatible with the "Invincible" sitting on her bilge and having *three* positions of stable equilibrium.

52. This being so, for the same reason the other empty-bottomed vessels must be deficient in stability when compared with that assigned them, and especially so the "Devastation," which is stated to have a deeper double-bottom than any of the above class of ships, while all ironclads must be deficient in consequence of the weight of their iron-armoured sides.

53. The late Controller of the Navy is reported to have said, that to wait for the result of the inclining experiments on the "Captain" before allowing her to go to sea would be Old Fogeyism. We have not been told in what sense this expression was used, but certainly the results as respects these ships fully justified the language, and imperatively demands an explanation.

54. The importance of a satisfactory explanation, not only as affecting the Royal Navy, but, with regard to the whole Mercantile Marine, can hardly be over-estimated, nor is it easy to over-estimate the responsibility of those who misrepresent the facts and throw impediments in the way of obtaining a solution of the difficulties with which the question has been surrounded.

55. We now propose to supply an explanation.

56. A floating body at rest displaces as much of the fluid in which it floats as is equal to its weight, and the fluid displaced is of the size and form of the portion of the body immersed. The measure, therefore, of the portion of the body that is immersed will give the measure of the quantity of the fluid displaced, and the weight of this quantity of the fluid will be the total weight of the floating body.

57. The amount of the floating body remaining above

water will be proportionally great as the mean specific gravity of the immersed body is less than that of the fluid in which it floats.

58. The resultant of the downward pressure of weight may be considered as collected in the centre of gravity of the system as the centre of the upward pressures may be considered as collected in the centre of gravity of the wetted surface or of the fluid displaced.

59. The body in passing to a state of rest will follow the law of least action, and will attain that state when equilibrium is established between the downward and upward pressures; and though these two forces may be equal and opposite, yet it is absurd to say they do not always meet, as is stated in *Naval Science*, "*in the same point.*" When disturbed from a state of rest by a force from without, a new state of things ensues.

60. Water being homogeneous, the centre of the figure of any volume displaced by a ship would be its centre of gravity.

61. If the immersed *portion* of the ship be also homogeneous, the centre of its figure will be its centre of gravity, and also, will coincide with the centre of the buoyant effort of the immersed body when upright, and when inclined also. If the volumes immersed and emerged by the inclination are also homogeneous and equal, and of like specific gravity with that of the body first immersed, the centres of gravity of the whole system and centres of buoyancy will be in the same vertical plane passing through the middle of the ship. If, however, these are unequal in volume and in specific gravity, then none of the centres will coincide, and the centre of gravity of the whole body will not be in the middle plane of the ship. Because these centres vary more or less with the inequalities and with the inclination, ships will be unequal in their stabilities and behaviour on one tack or side or the other, just what is sometimes observed by sailors to be the fact.

62. If, however, the immersed body be made decidedly heterogeneous, then its centre of buoyancy* may no longer

* On the contrary, by centre of buoyancy *here* is meant the centre of that

be coincident with the centre of figure as previously was the case, but may be above or below it, for the process of making the body heterogeneous was by concentrating the weights, and so increasing the specific gravity of some portions, thus decreasing their buoyant power; while this concentration reduced the specific gravity of other parts, it increased the buoyant power of these parts, and in proportion moved the centre of buoyancy from the position it occupied when the body was homogeneous.

63. For greater clearness, and that there shall be only one variable, it is assumed that the movement of the weights, above alluded to, is so adjusted that the centre of gravity is retained in the same position throughout the changes.

64. We may consider the further effect of making the immersed body thus heterogeneous, *i.e.*, some parts of very much greater specific gravity than others, and contrast the effect of this with that which obtains when the weights are equally distributed throughout, and therefore when the centre of figure and the centre of buoyancy were coincident.

65. The immersed body may be made heterogeneous in any direction, but if the two sides be *not* symmetrical, then the centre of buoyant effort of the body being carried over to one side or to the other, it will produce an inclination from the perpendicular, and thus reveal the defect.

We need, therefore, only consider the effect of the motion of the centre of buoyancy in two directions, those in the vertical line, one higher, the other lower than the centre of the figure.

66. We will first suppose the immersed portion of the body to be made heterogeneous by making the part above the centre of figure lighter than the part below the centre of figure without altering the height of centre of gravity. The centres of the immersed body now would no longer coincide; we should have the centre of buoyant effort of the body above the centre of figure, and more, we should have the centre of gravity of the immersed body below the centre of figure;

portion of the immersed body that is of less specific gravity than the water, and possesses buoyancy or floating power in proportion.

when the ship was upright in still water no effect would be perceptible.

67. We will assume the solids of immersion and emersion to be of like homogeneity with the upper portion of the immersed body; then, when the vessel was inclined, there would be an attempt of the light upper part to pass *up* into the perpendicular, and of the heavy lower part to return *down* to the perpendicular position it was pushed from.

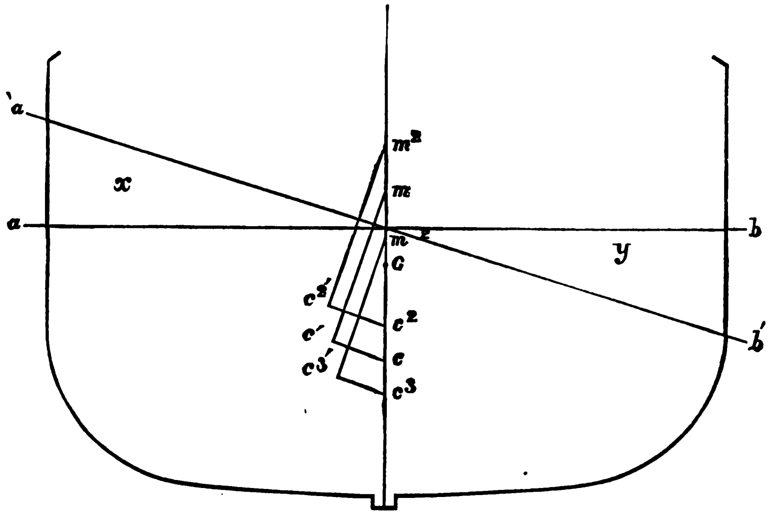
68. Or, to put it in another form, the centre of figure by the inclination would be moved out towards the lower side a distance proportional to the form and volume of the solids of immersion and emersion; but the centre of buoyancy of the body would also be moved out a like quantity, but its distance being measured off parallel but from a point higher up, would cut the perpendicular higher up, and to that amount would add to the stability, as deduced from the motion of the centre of figure.

69. We will now suppose the heterogeneity to proceed from the immersed portion *below* the centre of figure being made much lighter than that above the centre of figure.

70. The centre of buoyancy of the immersed body will be below the centre of figure, and the centre of gravity of the same would be *above* the centre of figure; while the ship is upright in still water this will be imperceptible, but when inclined the centre of buoyancy of the body would tend to rise, and the centre of gravity would have a tendency to go down, and to reverse the body, and this power would increase with the angle of inclination.

71. Or, in other words, when the vessel was inclined, by by this action the two centres would be moved out a like distance towards the lower side, but the centre of buoyancy of the body being *below* that of the centre of the figure, its distance would be measured off from thence, and would fall short of the line of pressure by a quantity; that quantity would be *negative*, and therefore must be deducted from the distance deduced from the position of the centre of figure.

72. Or to put it in another form. If the whole immersed



a b waterline when upright.

a' b' „ „ inclined.

x y solids of immersion and emersion.

G centre of gravity in each case.

c centre of buoyancy when body is homogeneous; c' the same when inclined.

c^2 centre of buoyancy when the body is heterogeneous from upper part being *lighter* than lower half; $c^{2\prime}$ the same when inclined; and m^2 is the metacentre.

c^3 centre of buoyancy when body is heterogeneous by upper part being *heavier* than the lower half; $c^{3\prime}$ the same when inclined; and m^3 the metacentre.

Buoyancy is the hydrostatic pressure that supports any body floating in a fluid. It is equal to the difference between the mean specific gravity of the floating body and the fluid in which it floats multiplied by the displacing volume.

When there are great differences in the buoyancies of the parts of floating bodies such as ships, the amount of these differences must be duly estimated and allowed for, or danger will ensue.

The four adjacent pages are taken from a previous publication as better explaining our meaning than the previous.

body, together with the solids of immersion and emersion, x and y, are homogeneous, then C will be the centre of buoyancy, and the centre of figure (on the hypothesis that these last two are identical), and if the vessel be inclined by a force to a given angle, the centre of figure and the centre of buoyancy will move out together to C′.

73. If the body be heterogeneous by the upper half, together with the solids of immersion and emersion, being of less specific gravity, and therefore more buoyant than the lower half, then C^2 will be the centre of buoyancy, and when the vessel is inclined to the same angle as before, will move out to $C^{2\prime}$, while C, as before, will be the centre of figure, and will move out only to C′.

74. If, on the contrary, the upper part of the ship be of *greater* specific gravity, and the solids of immersion and emersion so also because of the heavy armour, and the lower half of much less specific gravity because of the large empty spaces in the bottom, then the centre of buoyancy will be, say, at C^3, when the ship being inclined to the same angle, it would move out to $C^{3\prime}$.

75. If now perpendiculars to the water-line when inclined be drawn from $C^{2\prime}$, C′, and $C^{3\prime}$, respectively, these will cut the perpendicular when upright at m^2, m, and m^3. The power required to incline the vessel under the different conditions will be proportional to the distance of the above points m^2, m, and m^3, from G, the centre of gravity, and the respective stabilities will be in like proportion.

76. The arrangement of weights in which the centre of buoyancy is highest will have much the greatest stability, and that with the buoyancy lowest will be by much the least.*

77. That there may be an indefinite number of variations in the disposition of the weight, without necessarily moving the centre of gravity, is shown by a comparison of the "Captain" with other ships, and also an indefinite variation in the disposition of vacant space, which is but the converse,

* The movements of the weights is so adjusted as not to change the position of the centre of gravity.

also without changing the position of the centre of gravity, is also shown. Thus the "Captain" had the lowest centre of gravity of the following ships, yet she also had the largest proportionate amount, if not also the largest actual amount, of vacant space below the centre of figure:—

		Inches.
Centre of gravity of Achilles below load water line	..	18
" Captain "	..	39

78. The first case of heterogeneity explained above represented the condition of the old ships, and the latter case that of the new ironclads that have empty vacant space in their bottoms.

79. I have assumed above that the centre of figure is, as is asserted generally, also the centre of pressure of the water supporting the ship; which is true till motion commences.

80. In answer to our objection to the dangerous expedient of placing large empty spaces in the bottoms of ships, Mr. Reed wrote, "Everybody *knows* that a foolish architect might make " the vacant spaces between the bottoms dangerously large for " the size of the ship, and that the proper amount should be " determined by calculation;" therein admitting that the condition of homogeneity and heterogeneity was very real. He appears now to have retreated from that position, though true; surely, on so momentous a subject, a person who professes to teach "naval science" should, if he thinks he has found out his mistake, instead of ridiculing others for thinking as he once thought when he penned that passage, should confess his blunder, and offer proof that his new view is correct, viz., that the equal or unequal distribution of weights in a ship, supposing only the centre of gravity to be retained in the same place, makes no difference in the stability of a ship.

81. This he could not do while he continues to treat weights and buoyancies as different entities, in calculating the strains on ships produced by the unequal distribution of weights and buoyancies.

82. Mr. Merrifield wrote in the *Times* disparaging our views, and trying to stop inquiry, charging us with " count-

" ing a ship's weights twice, once being enough." Mr. Reed affirms his judgment, and yet these gentlemen ought to know that the calculations or counting of weights, made to determine the position of the centre of gravity, takes no cognizance of buoyancy, and still less of the centre of buoyancy, which was that of which we had written.

83. These gentlemen ought to know that the weights may be distributed to the sides or ends, or upwards or downwards, or may be on the other hand all centralized so far as space will admit, and yet the centre of gravity might be retained in the same point, and that these ends or sides will possess greater buoyancy in proportion as the weights are centralized. That the centre of buoyancy of the fore-body will move out from or move nearer to the middle transverse line of the ship as the weights are centralized or moved forward, and so likewise of the after-body and sides, and these weights and buoyancies, and not weights only, as Mr. Merrifield and Mr. Reed would teach, in fear of counting twice, must be taken account of whenever they are moved or first placed, if we would know the relative strains on ships' frames, &c., how a ship will set in the water more or less by the stern when launched, or when stowing or proposing to stow the cargo, or how she is likely to behave, safely or otherwise, when she goes to sea.

84. Will these gentlemen say that a change of law takes place when a distribution of weight in the vertical plane is made or considered, and that the centre of buoyancy does not move up or down from the water-line when weights in the immersed body are moved down from the water-line or up from it, just as truly as it moves horizontally in the cases mentioned and illustrated in every day experience?

85. They may say we admit that moving weights horizontally will affect a ship statically, her frames may be strained and her trim and behaviour changed; and, further, we admit that if we move the weights vertically or horizontally we shall affect the ship dynamically; but we ask, " How can you " affect the ship statically by moving weights vertically in " immersed portion of her body?"

86. The following illustrations and arguments will establish that there is no change of law such as would be required if these gentlemen's views were correct, and that the moving of weights in the vertical plane may make a more radical because more dangerous change, than moving weights in the horizontal plane.

87. We will suppose the specific gravity of the water in which a series of bodies float to be 1,000.

88. We will suppose these bodies to be made up of six sections in each of exactly the same dimensions, and each body to be of the same total weight, but, except in Fig. I, the sections to differ in the specific gravities by pairs and be of the amounts marked on each.

89. These bodies to be so arranged that they all float at the same depth and with their centres of gravity all in the middle of the load water-line.

90. The mean density of each body with its six sections is half that of water, therefore they will all float with half their body out of water. G, the centre of gravity of each, will be similarly situated in all in the load water-line *a b*.

91. We will assume, according to the received opinion, that C in the centre of figure is the centre of upward pressure.

92. Now, if we remove section 6 from Fig. I, the equilibrium of the remaining portion will be undisturbed, as G will move to G′, and C will move an equal distance to C′, while the remaining body will preserve the original load water-line *a b*, and this will be so, because when we remove section 6 we remove an equal amount of buoyancy and of weight, and therefore equals remain, and balance each other. This Figure illustrates the condition of homogeneity when it obtains in a ship, and the only condition discussed by the great writers on naval architecture, and the only condition to which their metacentric method can be applied with any approach to safety.

93. We now take Fig. II, a heterogeneous body of the same weight as Fig. I, as though weight is taken from sections 2, 3, 4, and 5, it is added to 1 and 6, and though it is made up

of sections of unequal densities these are distributed in pairs of equal density at equal distances and opposite sides of the centre, and so also as to buoyancies, they are equally distributed on opposite sides of the centres of the immersed figure, so they also balance each other.

94. While the floating body was of uniform density we could with safety deal with the question in the aggregate, and be content with the abstract fact, that water being of uniform density one foot gives out an equal buoyant effort with any other, and, therefore, that any one foot of the body would be equally *supported* with any other; but *now* that the densities of the different parts of the floating body, or congeries of bodies, are unequal, we must consider the practical fact, whether and what parts are or are not water-borne, *i.e.*, what is the amount and direction of the resultant forces on each part, both of buoyancy and of weight?

95. For though the pressure of each foot of water similarly situated will be the same, yet the degree in which any portion of a body, floating on it, will yield to that pressure will depend on the specific gravity of this portion.

96. If the weight in any given section is greater than the buoyancy or displacement of that section, then the resultant force in that section is more or less downwards; if, on the contrary, the buoyancy of any section is greater than the weight of that section, then its resultant is upwards; nor is this matter of opinion, but is a fact recognized in various ways.

97. In Fig. II the resultants of 1 and 6 would be downwards. In other words, did this figure represent a ship lengthwise, the weight would be borne up by the middle sections and the two ends would droop, exercising a breaking strain equal to the deficiency of buoyancy and their distance from the middle, producing the condition in a ship known as "hogged."

98. Line-of-battle ships before the introduction of Sir R. Sepping's system of trussing, used to break their sheer, or hogg 13 inches on being launched.

99. If we imagine the sections at once to separate, then 1

and 6 would sink, 2, 3, 4, and 5 would capsize, and would arrange themselves with their longer axis horizontal.

100. If we remove section 6 from Fig. II, as in Fig. I, the equilibrium of the remaining portion will be destroyed and it will capsize, the resultant force of 1 being downwards and greatest; the body will adjust itself with that section horizontal, and at the bottom, section 5 being uppermost, and the body from losing the great buoyancy of No. 6 will sink deeper, and its centre G will no longer be in the load water-line.

101. The cause is the following in removing section 6, a large amount of weight is removed, therefore, the centre of gravity of the remaining portion is moved very much to the left, so that the resultant is moved into section 1 and downwards, while as little or no buoyancy was removed, its centre remains the same and resides in section 4, which is *upwards*, so these two, 1 and 4, the resultants acting as a couple rotate the body to the left, and till the centres place themselves again in a vertical but at right angles to the former vertical when at rest.

102. If now we examine Fig. III, where the arrangement of weights is inverted, we shall find that the resultant of the two ends is *upward*, and the two centre sections downwards, producing the condition formerly experienced in steamers with their great weights in the middle, making the resultant of these sections to be downwards, producing the condition known as "sagging."

103. If we remove section 6, as in the other cases, the body will capsize, but now in an opposite direction, and the body will adjust itself with section 1 uppermost as it will be the lightest, because in the operation of removing 6, we have taken away a great amount of buoyancy but very little weight, therefore, the centre of gravity has moved very little to the left and residing in section 3, the resultant of which is downwards; but the centre of buoyancy is moved to section 1, the resultant of which is upwards, these two forces act as a couple, and rotate the body till these centres adjust themselves in the vertical line, which will be when the position is at right angles to the former.

104. The above stated results establish beyond question that enormous differences are produced by an unequal distribution of weights and buoyancies, for if the specific gravity made no difference, if only the centre of gravity were unmoved, then the removal of a section of small specific gravity from Fig. II, or a heavy one from Fig. III, would have made no more difference than the removal of a section of the same size, but of a uniform specific gravity with other sections in the body from Fig. I.

105. If these facts be considered in reference to their influence on stability, we shall find that their effect is equally great.

106. In Fig. I, which is of uniform density, the resultant of every section and every part of every section is the same, differing only in power greater in proportion than the distance from the centre of gravity or centre of rotation.

107. Not so in Figs. II and III, where the resultant of every section on each side differs from that of every other section on the same side both in amount and it might be in the direction also.

108. We have seen the enormous power of sections 1 and 6 in supporting Fig. III, with the total want of power in 1 and 6 in Fig. II, these having to be largely supported by other sections, so that were it not for this support they would sink,—which condition must exert an influence for good or for ill, giving support or causing a burthen wherever placed in water.

109. Thus, if Fig. II be inclined as in Fig. IV by a force from without, then as the specific gravity of sections 1 and 6 is very great, and the resultant force downwards, the immersion and emersion will not only not give any support, but will detract from the support given by other sections on which this form will be entirely dependent, this distribution of weights will have very little stability. The less that the sections 3 and 4 are so little distant from the centre of rotation, and their volume so small; even with a large angle of inclination, with this distribution of weight, the stability is not only much less than that assigned to it by the present

and 6 would sink, 2, 3, 4, and 5 would capsize, and would arrange themselves with their longer axis horizontal.

100. If we remove section 6 from Fig. II, as in Fig. I, the equilibrium of the remaining portion will be destroyed and it will capsize, the resultant force of 1 being downwards and greatest; the body will adjust itself with that section horizontal, and at the bottom, section 5 being uppermost, and the body from losing the great buoyancy of No. 6 will sink deeper, and its centre G will no longer be in the load water-line.

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109. Thus, if Fig. II be inclined as in Fig. IV by a force from without, then as the specific gravity of sections 1 and 6 is very great, and the resultant force downwards, the immersion and emersion will not only not give any support, but will detract from the support given by other sections on which this form will be entirely dependent, this distribution of weights will have very little stability. The less that the sections 3 and 4 are so little distant from the centre of rotation, and their volume so small; even with a large angle of inclination, with this distribution of weight, the stability is not only much less than that assigned to it by the present

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102. If now we examine Fig. III, where the arrangement of weights is inverted, we shall find that the resultant of the two ends is *upward*, and the two centre sections downwards, producing the condition formerly experienced in steamers with their great weights in the middle, making the resultant of these sections to be downwards, producing the condition known as "sagging."

103. If we remove section 6, as in the other cases, the body will capsize, but now in an opposite direction, and the body will adjust itself with section 1 uppermost as it will be the lightest, because in the operation of removing 6, we have taken away a great amount of buoyancy but very little weight, therefore, the centre of gravity has moved very little to the left and residing in section 3, the resultant of which is downwards; but the centre of buoyancy is moved to section 1, the resultant of which is upwards, these two forces act as a couple, and rotate the body till these centres adjust themselves in the vertical line, which will be when the position is at right angles to the former.

104. The above stated results establish beyond question that enormous differences are produced by an unequal distribution of weights and buoyancies, for if the specific gravity made no difference, if only the centre of gravity were unmoved, then the removal of a section of small specific gravity from Fig. II, or a heavy one from Fig. III, would have made no more difference than the removal of a section of the same size, but of a uniform specific gravity with other sections in the body from Fig. I.

105. If these facts be considered in reference to their influence on stability, we shall find that their effect is equally great.

106. In Fig. I, which is of uniform density, the resultant of every section and every part of every section is the same, differing only in power greater in proportion than the distance from the centre of gravity or centre of rotation.

107. Not so in Figs. II and III, where the resultant of every section on each side differs from that of every other section on the same side both in amount and it might be in the direction also.

108. We have seen the enormous power of sections 1 and 6 in supporting Fig. III, with the total want of power in 1 and 6 in Fig. II, these having to be largely supported by other sections, so that were it not for this support they would sink,—which condition must exert an influence for good or for ill, giving support or causing a burthen wherever placed in water.

109. Thus, if Fig. II be inclined as in Fig. IV by a force from without, then as the specific gravity of sections 1 and 6 is very great, and the resultant force downwards, the immersion and emersion will not only not give any support, but will detract from the support given by other sections on which this form will be entirely dependent, this distribution of weights will have very little stability. The less that the sections 3 and 4 are so little distant from the centre of rotation, and their volume so small; even with a large angle of inclination, with this distribution of weight, the stability is not only much less than that assigned to it by the present

metacentric mode of calculating, but very much less than that given by the distribution of weight illustrated in Fig III, where the resultants of 1 and 6 are very great and upwards, as will be seen by Fig V, in which a great force will be required to immerse a greater quantity of No. 6 section; owing also to its greater volume as well as small specific gravity and great distance from the centre of rotation; also as the position of the body was originally much dependent upon section 1, an additional force will be required to draw it out of the water. So from these causes the stability with this arrangement of weights and buoyancies will be vastly greater than that assigned by the present metacentric method which supposes a uniform density throughout.

110. Thus we have seen that spreading the weights out unequally and largely to the sides reduces in that proportion the statical stability much below the ordinary estimate; on the other hand, doing so in that proportion increases the moments of inertia of the sides; so that when ships are in the midst of small waves they will remain comparatively undisturbed by *them* and easy; the sides being little buoyant, the waves will tend to break over or against them, not having sufficient power to lift or disturb the ships, and then relative easiness will obtain, irrespectively of the height of the centre of gravity.

111. But this inertness under such circumstances will obscure the want of statical stability, and ships which have this arrangement of weights will be reported of, much to their danger, by those who are unable to explain the symptoms, because of this relative easiness and inertness and obscurity, as possessing a sufficiency of stability; and further it will be reported that this easiness is the result of a high centre of gravity, when we have seen it may occur quite irrespective of the height of the centre of gravity.

112. But when high seas arise all will be changed; the want of statical stability will manifest itself, and if speed be not given to such ships for the purpose of obtaining for them stability of another kind, they will tear themselves to pieces, and if worse weather and seas supervene, they will roll

deeply, dangerously, and possibly *over*, proving the inertness to be absence of vital power.*

113. If we place Fig. II in the water with its sections horizontal, as is in Fig. VI, the centre of gravity will still be in the load water-line *a b*, and, though there is much top weight, yet will the stability be considerable; for suppose this inclined as represented by Fig. VII, a further portion of section 3 must be immersed; but this cannot be done without the application of a considerable power, because of its great buoyancy and the distance of its centre from the centre of gravity.

114. Moreover it will be difficult to emerge any portion of section 4, bearing as it does so much of the whole weight of the body, the more difficult that the centre of buoyancy of the whole body is so high.

115. We may assume that, as each section is in itself homogeneous, when any possess buoyancy, its centre will be somewhere below the middle of each section, but as section 6 is without buoyancy—for it would sink if left unsupported—and as section 5 possesses little buoyancy as compared with that of section 4, the mean centre of buoyancy must reside within the limits of this last section, say at C and C′; therefore the distance that the centre is moved by the inclination must be measured from this point thus high, which will place *m* high and give a large metacentric height, and therefore considerable stability.

116. On the contrary, if Fig. III be placed in the water with its sections horizontal, as in Fig. VIII, it is doubtful whether it would remain so for a moment if left to adjust itself, and this because the centre of buoyancy would be so low, and the section near the load water-line being without any buoyancy, and therefore not giving any support; for suppose that model inclined and held so for examination, sections 3 and 4 have no buoyancy, therefore little power will be required to immerse or emerge portions of them. Then section 4 has little buoyancy as

* This will be the fate of the "Devastation" if ballast or other weight be not placed in the spaces between her bottoms.

compared with section 6, and therefore the mean centre of buoyancy must reside in the latter section, say at c; because of the small value of x and y, the distance that c will be moved by the inclination will be small, say to c', a perpendicular from which would intersect the original perpendicular below the centre of gravity, and hence it is that the body with such an arrangement of weights and buoyancy must capsize.

117. A recent experiment—the copy of an official report on which is given in Appendix B—gave practical proof of the correctness of these views and those which we have long contended for.

118. Five hundred weight of cork was placed round the out- and in- side, near the gunwale, and under the thwarts and stern benches of a 30-foot ship's cutter. She was then filled within three inches of her gunwale with water, and yet she bore up 40 men easily—14 men standing on the gunwale, hanging over by ropes attached to the opposite side, capsized her with difficulty. After she was capsized the men, catching by a batten on her bottom, righted her instantly, and the 14 men got into her again.*

119. A similar cutter, but without cork, when filled with water sank with less than 14 men, so that they had to swim to save themselves. She was capsized by seven men, but could not be righted so that the men could get into her, as she turned over at each attempt.

120. It will be observed with what difficulty the first cutter was capsized, because of the power of the cork on her topsides, and the facility with which that cork came to the surface when a slight inclination was given to the boat, and the persistent way it sustained, without oversetting, 40 men in it, though this involved the necessity of a great amount o. top weight.

121. The condition of this boat is the converse of our ironclads.

* Every merchant ship ought to be compelled to carry a boat of this description, to carry her crew comfortably, and fitted with tins of bread, preserved meats, and water; and the quarter-boats of men-of-war should be fitted with cork.

122. It would be a valuable experiment to take a third cutter, and place even a smaller quantity of cork near her keel, with a band of iron round her above of equal weight, it would be seen with what facility she would be capsized, and the danger in which some of our ironclads are, the condition of which she would then represent, and, while pains are justly taken to protect the lives of boats' crews, plans are adopted that place the lives of whole ships' crews, with valuable public property, in grave peril.

123. Cork has been long in use for making life-boats, but that which gives the peculiar value to Sir Wm. Hall's arrangement, is that all the buoyancy is placed above and mostly at the sides above, this makes the boat extremely stiff and difficult to capsize, and it facilitates her being righted when she has been capsized, whereas in many life-boats a proportion of the buoyancy is placed low down, this tends to raise the weights and the general centre of gravity, facilitating a capsize and when capsized making her more difficult of being righted.

124. During the Kaffir War of 1850, the troops were cut off from communication with the Cape Colony except by sea, the writer designed surf-boats to facilitate keeping open this, but it was thought desirable to try other designs than his, we predicted that such would capsize, stating our reasons: one was built and taken up by us to Buffalo Mouth, we passed on to Mauritius for a regiment, on our return there weeks after, notwithstanding that the dangerous features in her had been materially reduced, we found that this boat had turned over and drowned her crew.

125. Her peculiarities were an empty space between an inner and an outer bottom and a high centre of gravity!

126. We commend this and the unhappy fate of the Captain's crew to those gentlemen who persist in advocating, neither wisely nor well, deep empty spaces in the ironclads' bottoms and high centres of gravity!*

127. We have been told that the spaces between the

* The centre of gravity of the "Captain" was lower than that of any ship in the Navy!

bottoms of ironclads were not made deeper for the purpose of raising the weights. But it must be admitted that Mr. Reed ought to know what was the intention of the Admiralty Constructors in making those spaces so deep. He said, in his Lecture at the Royal Institution, February, 1871, "The distance between the double bottoms has been made great in recent ironclads, expressly to facilitate the raising the engine's boilers and other weights, because it has been ascertained that the tendency of ships to roll has been reduced by this means."!!

128. In these various figures, we have shown that the degree of distribution or concentration of weight may be extremely diversified, both vertically and horizontally, and yet that the centre of gravity may be at the same height in all, proving that its position by no means defines the degree of dispersion or of concentration, and moreover, that its determination neither determines the position of the centre of buoyancy, nor does it show the great evil of undue dispersion of weight.

129. We have also shown that the reasoning in respect of the distribution of weight in the vertical plane that is admitted to obtain when considering the effect of the distribution of weights and buoyancies, longitudinally and transverse horizontally, must equally hold, unless there be a change of law, but this cannot be admitted without proof, and cannot be assumed with reason or without danger.

130. Those mathematicians who devised the metacentric method, expressly state that it was only applicable to small angles of inclination and to homogeneous bodies.

131. Therefore, to apply it to ironclads with an extreme concentration of weight at their sides is erroneous, and it assigns them an amount of statical stability that they do not possess, and this is aggravated in those that have large empty spaces in their bottoms, as they have proportionably less stability and are even more dangerous.

132. This mode of calculation was always erroneous, but less so in respect of the old ships as they had heavy bottoms, and their ballast and heavy weights were down on them,

while they possessed buoyancy near the load water-line, thus the old ships of the line, notwithstanding their top-hamper, possessed a considerable amount of stability, and their distribution of weights and buoyancies were analogous to those in Fig. VII, as is also the case of the French "Océan" class.*

133. While those in such vessels as the "Monarch," the unfortunate "Captain," and "Devastation," and the "Invincible" class, and "Sultan," before they were ballasted the distribution was more analogous to that in Fig. VIII, differing only in degree, and though the "Devastation's" centre of gravity be lower than that of the "Monarch," nay, even lower than that of the "Captain," and though she may possess greater proportionate beam, yet if, as we suppose it is, that the empty space in her bottom is proportionately greater, and the weight on her sides proportionably greater also, she may approach far too near to the dangerous arrangement illustrated in Fig. VIII, to justify her being navigated without ballast. True it is proposed by some that she shall be subjected to a test that shall set at rest, for ever, all question as to her stability and safety, forgetting that this cannot be done without capsizing her.

134. The "Captain" was tried in a gale and was pronounced to be perfectly safe, however, on the night she was lost she went over to 18°, and no doubt returned from that, and the gunner, who was scanning her behaviour with a critical eye, said, "I have a good ship under me," but the opinion was hardly formulated, if so much, when she went again over to 18°, but this time never to return; thus, we have in her case, and why not in that of the "Devastation," half a degree or so between what this man thought to be a crucial test and total loss!

135. And when she was lost it was found to be more convenient to discuss, rather than the proper question, whether if she had had a higher side, and the *reserve* of stability it might have given, she would thus easily have capsized?

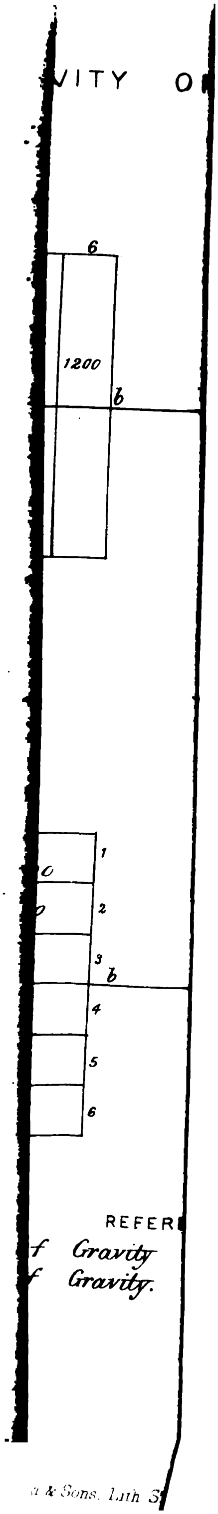
* In consequence, the errors of calculation are as numerous and as great as are the differences in the degree of the distribution of weight.

136. If she had had the amount of initial stability which the present mode of calculation had assigned to her, she would not have gone over to the first 18°, every property would have been improved, and the ship and crew saved, because the reserve of stability would have been where it ever ought to be, in the excess of initial stability over all inclining forces, whencesoever they might come; but she did not possess by one-half at least as much initial stability as was supposed, or as was necessary, and it is dangerously delusive to talk of increasing the safety of the "Devastation" by the addition of a reserve of stability from an increased height of topsides, when her initial stability is deficient, and perhaps one-half less than is supposed, and this deficiency could not have been otherwise than increased by the addition of deck-load in the shape of topsides! and it is infinitely more useful and convenient to discuss this question now rather than when she is lost.

137. In placing large quantities of cement ballast in the "Sultan," "Invincible" class, "Inconstant" and "Tamer," and in others that have been reported, not only is the centre of gravity *lowered* many inches, and the stability increased to that extent, but the centre of buoyancy is *raised* at the same time, and in proportion as these two centres are brought nearer to each other, the metacentric height will be raised by these operations, and the stability increased beyond that which is generally supposed, but nevertheless proved in the "Sultan" and "Invincible" to be the fact, by their greatly reduced angle of inclination when turning at the measured mile.

138. If the legislation that may be recommended by the Royal Commission appointed to take evidence as to the condition and equipment of the vessels in the Mercantile Navy, shall be based on the supposition that the existing methods of determining the stabilities of ships is correct, very lamentable results will follow, for Parliamentary protection and warrantry will be sometimes given to ships in a very dangerous condition.

139. We can have no doubt that many vessels that have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been lost have been so, not that they were not well constructed, not sound, not well found or were subjected to unusual gales or seas, but simply because they were improperly stowed, a French architect has said, "That a bad ship badly "stowed was very bad, but that a good ship badly stowed was "no better."

140. We have shewn that it was quite possible to have the centre of gravity at the same height in two ships, and yet that one should be unsafe while the other was quite safe, and, therefore, that the position of the centre of gravity, even if known, which it is not, alone is no safeguard.

141. We have shown that of two vessels of exactly the same form and dimensions, one may be unsafe and the other safe, quite irrespective of the height of freeboard, both being of the same height.

142. It may be shewn also that it might contribute to the safety of a given ship, under certain circumstances, to immerse her more, nay, even this by adding a deck-load, yet not in the shape of an increase of topsides. Therefore, to fix the water-line by law would be but a snare.

143. The only sufficient ground of safety for the lives of our seamen, and for the property of the country, which it practically becomes by insurance, is to have at each port qualified Stevedores to instruct the men how to stow the various cargoes. Moreover, no captain or mate should receive a certificate of competency until he had passed an examination in the principle to be observed in stowing ships' cargoes.

144. But before that, or in order that the Stevedores shall be properly instructed, the positions we have established as to the mode of calculating stabilities will require to be accepted, for otherwise a Stevedore might stow a ship in strict accord with the present calculations, and suppose her safe, when she might not be more safe than "Invincible" class or the "Sultan" before they were ballasted, or the "Captain" when she capsized!

* The exact distribution of weight in each ship need not be defined, but limits of safety might be determined, and these should not be allowed to be passed.

145. If this course is not adopted unwise legislation will follow, naval architects and shipowners will be relieved from instead of being brought under more distinct responsibilities in respect of the lives of the crews, and an injury instead of a benefit will be done.

146. Naval architects should be held responsible for the strength of their constructions and for the correctness of their designs so far as safety is concerned, and for the correctness of the data they supply to the shipowner, and they should be obliged to supply complete drawings, shewing the position the centre of gravity would occupy, on the supposition that the ship were light or loaded to her load-line with a cargo uniformly distributed. And if cargo was not attainable, the weight of ballast that would be necessary to make her safe under sail at sea.

147. The height of the centre of gravity on those conditions should be marked on the ship amidships, for without this it will be impossible safely to distribute the cargo.

148. Thus we have seen that if the weights are unequally distributed to the sides, in that degree the ship loses stability, therefore her centre of gravity should be proportionably lowered.

149. Where men's lives are at stake, any defects in the above, whether arising from ignorance or from faulty design or from erroneous calculations, are crimes.

150. We did not approve of Sir Wm. Symonds's designs, but if he did no more for the Navy, he established, at great cost to the country, that ships' sails were effective, and therefore the ships themselves, in proportion as ships were endowed with stability to stand well up under the pressure of their sails; but in our day, ships have been beyond all parallel denuded of this stability, so that they could not stand up under a moderate pressure of wind on the masts without any sail whatever.

151. Had the exigencies of the Franco-German war required that we should have sent troops to Belgium, the "Invincible" and her sisters perhaps would have been sent to sea with, say, 2,000 soldiers in each, without having been

previously tried, when with this small additional top weight, if sail were put on them, and they to have been taken in a squall, they would have capsized and have probably drowned every soul.

152. Yet the men responsible for these things have misled the public by reports of the performances of the ships under sail, as if these took place under the circumstances in which they were designed to be, without ballast, whereas they had had their sails reduced and 360 tons of ballast placed in them, while from 70 to 200 tons of ballast were found sufficient for old ships, though they carried four tiers of guns, and much greater masts and a greater spread of sail.

153. If an Act of Parliament shall define a ship to be safe if only certain ill-judged conditions are complied with, it will be found that ships' crews have been drowned according to Act of Parliament.

154. We have seen that one of our ironclads was capsized from not having been endued with sufficient stability that six others were pronounced to be "unseaworthy," til they had been ballasted to the extent of 350 tons, and thei area of sail reduced, that an eighth was deemed unsafe till ballasted to the extent of 500 tons nearly, and we are informed that other ironclads are to have quantities varying up to 450 tons.

155. We have seen that the designs of all ironclads are unscientific, and that the calculations made concerning them are inaccurate and have led to the state of things described above, and that some of them are even now unsafe. Admiral Paris, of the French Navy, said, in 1869, "the new broad-" side ironclads are worse rollers than the old ships used to be, " and, even in moderate weather, bring their guns under the " wave." Yet we were told that science had shown it to be a popular fallacy to suppose that armour would make ships roll more; charlatanism had so determined, not science, but the error was not discovered till great damage, at great cost, had been done to our ships.

156. In bad weather these ships tear themselves to pieces unless driven at considerable speed, if they are not

unsafe also then, and it is obvious that when they are so driven it must entail great wear and tear besides great cost for coals, that might be much better employed, moreover, that all are unsafe in bad weather when near the shore unless they are kept under steam at great cost.*

157. We know that the cost of making these ironclads is very great, and that the cost of the increased dimensions to enable them to carry this great and exceptional weight of armour is still greater, while the papers have told us of accidents that arose from their unmanageableness, together with the facility with which their sides have been penetrated by their contact one with the other.

158. But why all this damage and expense? Simply because we have been seeking to do that which is unattainable, *i.e.*, to keep out all shot and shell, the more hopeless that we have already exceeded the limit of weight of armour that is consistent with a safe and generally efficient ship, while we are yet very far from having attained to the maximum limit of velocity and of penetration.

159. If the plan of guns suggested by that eminent mechanician, Mr. Bessemer, be adopted, it will no longer be a question of mere penetration of the iron coat and an admission of the shot or shell through this hole, for the shot he proposes will drive so much of the side in as to admit the ship "from which it is fired to pass in also."

160. To insure hitting, Mr. Bessemer has devised elegant instruments for firing with accuracy of time and of elevation, objects more important for ships and more difficult of attainment because of the motion of their gun platforms. Then, for measuring distances, we have had descriptions of valuable telemeters in successive numbers of the engineering newspaper, and to these Mr. Bessemer has added another, which gives reason to hope that a completely effective instrument will soon be obtained.

161. True, the proposition of Mr. Bessemer is to throw a 5-ton shot at a velocity not less than 1,000 feet per second, but so elastic is his system, that by greater velocity he

* See Admiral Sir Thomas Symonds's report on this subject—Blue Book.

might obtain almost as great results with much smaller shot, but indeed such effects as he could produce with his great guns are unnecessary for a gun to be used either in ships or against ships, for less than half the weight of such shot striking the turret, say of the "Devastation," or on her armour abeam, if those did not smash up quickly, the ship herself would be rolled over.

162. Some may imagine that such guns are too large for general use, but this objection could not lie against them for batteries and gun-vessels for harbour and coast defence, and they would be infinitely less expensive, and, being movable, more effective than fixed forts.

163. In the contemplation of such guns, and his are not the only plans that would admit of a higher than the usual velocity, and consequent destructive power, it seems somewhat more than waste of public money to build such unwieldy ships to carry heavy armour at once so expensive, so damaging to their qualities, and yet so useless.

164. Is it not preposterous to build such ships as the "Devastation," of 9,000 tons, costing the greater part of half a million of money, to carry only four guns, ships unfit for general service, drawing too much water for effective coast defence, for armies are not landed under beetling cliffs, where the water is steep to, and suitable for such ships, if there should be *space* for them, but upon beaches where the water is shoal and where such ships could not approach.

165. The increasing facilities for using the torpedoes effectively makes it still more unwise, thus putting the greater part of half a million upon one throw.

166. Then, because of the enormous cost of these great ironclads, our Navy is being reduced to numbers that a child might count, and because of these limited numbers, the national interests are unprotected at some points inviting attack.*

167. Would it not be wiser at once to abandon the vain attempt to keep all shot and shell out, but instead, to allow

* Several of our little but expensive wars have been the result of this parsimony applied to Navy or Army.

all to pass as formerly, seeking now only to make our ships as unsinkable as might be consistent with general efficiency, and which for practical purposes would be absolutely so.

168 Every one who has tried knows full well how almost impossible it is to sink a Chinese Junk, because of her numerous compartments, and iron affords greater facilities for subdivision and tightness than the material of which they are built.

169. Three unclad ships made nearly unsinkable by compartments, and by the use of uninflammable light wood, might be built for less money than the "Devastation" cost; vessels that could traverse the ocean safely and inexpensively, adapted to all the exigencies of general service, and better suited for coast defence, because of their greater number, smaller size, and lighter draught of water.

170. While employed to engage such as the "Devastation," they would hunt her down and destroy her in more ways than one, for they might carry more than 10 guns for one of her's, and while she was endeavouring to destroy one, the others would destroy her.

171. There is no reasonable doubt that if one of them of only 1,000 tons, moving at 10 knots or 17 feet per second, were to stem the "Devastation" on her broadside near the turret, she would roll her over, and the light bottom of that ship would contribute to the catastrophe.

172. It should not be lost sight of, that the tendency of the employment of ships of doubtful character, is to destroy the confidence of our men and with it the moral force. A man said to me the other day, "they allowed us, sir, to " travel the world over for three years, and then heeled the " ship and found her unsafe and put ballast into her."

173. It strikes me, that if the helm of the "Devastation" had been put over when that large body of water was tumbling in over her bows, as was lately described by the papers, she would have been in considerable danger of being turned over by the centrifugal force and the increased weight of water, which would then come in over the lower side of the bow, combined.

174. Nor should it be overlooked that fighting behind armour tends to deteriorate the character of our men, while it forfeits the advantages of the prestige and pluck of our nation, without any compensating advantages, and might well, for every reason, be abandoned, as it never ought to have been adopted, at least, by us!

175. Then the unmanageableness which the armour entails, deprives our men of the opportunity of exhibiting the dash at and celerity of movement which so distinguished them, and was a prime element in their past successes, while it lays them open to damage or defeat by an enterprising enemy who is not so unwisely armour-bound.

176. As if to facilitate the action of our ships in running into each other, or running under water, they have long projecting snouts under water, increasing their unhandiness, but given on the plea that below water is the best place to ram an enemy; this is a mistake, for striking below the centre of gravity would turn the enemy the wrong way and she would fall on board the assailant, entangle and endanger her, while if she were hit above the centre of gravity she would turn over and go down, or the side would go in, but not so as to entangle the assailing vessel.

177. After the foregoing was in type, an experiment on the "Devastation" and "Sultan" was made, and as the results accord with our argument, and so far tends to establish our views, the facts, with a discussion, are added.

178. It had been said that the centre of gravity of ironclads had been raised to "check-rolling," that raising this centre lessened the "tendency to roll," and also that "the "effort of stability is the lever by which a wave forces a ship "into motion." That, "if a ship were destitute of this "stability no wave that the ocean produces would serve to put "her in motion, whether the stability be due to *deeply stored* "*ballast*, or to the broad plane of flotation:" and irrelevant facts were offered in "Our Ironclads," to prove that when the centre of gravity was higher the arc of roll was smaller.

179. We had asserted, on the contrary, that all experience proved, that raising the centre of gravity, never reduces the

arc through which a ship rolls, but the opposite, and that every ship rolls through larger arcs as she lightened by the consumption of wate·, coals, &c.

180. It had been said that "the danger of overturning "becomes greatest when a ship lies in the trough of the sea "amongst waves, whose periodic time from crest to crest is "equal to the natural periodic time of a double roll of the "ship, for then there is a tendency through the coincidence "of the successive impulses given by the waves, with the "successive rolls of the ship to increase indefinitely the "angle of roll,* therefore (it was added) every ship, to be "safe, ought to have a very considerable period, and in order "to do that she must have very little initial stability."

181. This was proposed to be effected by raising the centre of gravity, and by distributing the weights as far as possible towards the sides.

182. The last condition to a great extent followed from the use of heavy armour; the first was so effectually applied to the "Invincible" class and "Sultan," that this double depletion of stability nearly proved fatal to them, to save them, from 350 to 500 tons (nearly) of ballast was introduced into these seven ships, as was, curiously, alleged to place the centre of gravity where it was intended it should have been, but, as was stated by Mr. Goschen in the House, to *correct a mistake made in their construction.*

183. Nevertheless, we will accept the present condition of the "Sultan" as that dictated by a mature judgment, and examine it, to ascertain what measure of satisfaction the recent experiment affords as to her safety and general efficiency.

184. As to the amount of her stability, or any other ship's, little is known, as the calculations made to determine them proceed on the erroneous hypothesis that there is a uniform distribution of weights and buoyancies throughout the ship; that the body of the ship including the solids of immersion and emersion are homogeneous yet it never is so, and in these armour-clad empty-bottomed ships it is far otherwise.

* See Froude, Trans. I. N. A., 1862.

185. The "Devastation" and "Sultan," two ships of a little more than 9,000 tons weight each, were rolled in smooth water by racing men from side to side at the end of each roll,—the "Devastation" by 400 men, the "Sultan" by 500.

186. The angle of double roll reached by the "Devastation" was 14½°, and it was stated that she could not be forced beyond that, at least by the force used, while the angle reached by the "Sultan" was 29½° or 15° from the perpendicular, after many races of the men across her deck, at which time an order was given to the men to cease running, loose things about the decks and gear, not having been specially secured; for no one, it is added, anticipated such liveliness on "Sultan's" part.

187. It is asserted that the "Sultan's" period of double roll was 17·7 seconds, and that of the "Devastation" 13·6, and it is asserted that as these periods, especially the longer, represent lengths of wave that almost never occur, assurance is drawn that there is no danger of either these ships being rolled over, but in this is overlooked, what is notorious to those acquainted with the sea, that solitary waves of very long period and great magnitude occur, and pass along without interfering with the ordinary waves, and which would make short work with a ship caught at an extreme roll, or striking a ship high up, that has only small stability, like the "Sultan."

188. We must consider what the factors were that occasioned the "Sultan's" roll to be double that of the "Devastation."

The former was 45 feet longer; this vertical area would tend to limit her angle of roll; but the latter had bilge pieces which tended so far to place them on an equality.

189. There were 500 men in the former ship, and only 400 in the latter, but then the "Devastation" is 3 feet wider, and this is increased by the considerable tumble home of the "Sultan" at her upper deck, therefore, the leverage through which the weight acted would be much less in the "Sultan."

190. No doubt the height of the "Sultan's" upper deck,

where the men raced, was much higher, 10 feet or more, than that of the "Devastation;" but this could not affect the horizontal leverage, but as the men in each case had to run up hill, those in the "Sultan" would act through a greater lever in the amount of the greater height, to retard quickness of motion, and to limit her arcs of roll.

191. It is clear, then, that none of these differences could have produced the result witnessed, and that there must have been another and a ruling factor; this we find in the difference in their metacentric heights.

192. The centre of gravity of the "Devastation" was inconsistently placed much lower for safety, because she was a low freeboard ship, and therefore would not possess the reserve of stability that a high side might give as compared with a low side. Inconsistently, we say, for they placed her centre low, while they were contending that the effect would be to make her roll more, and possibly roll over.

193. The centre of gravity of the "Devastation" was lower than that of the "Sultan," to which fact the masts of the latter contributed, and her metacentric height and stability were greater and greater still than the estimate, because of her low side and the greater concentration of her weights towards the middle line of the ship; therefore, instead of its being the fact that the ship with the higher centre of gravity and less initial stability was least affected by these successive impulses from the passage of the men from side to side, somewhat analogous to the impulses of recurring waves—as might have been reasonably expected, the ship, with the higher centre of gravity and less stability, was rolled through double the arc that the ship was that had the lower centre of gravity and greater initial stability. How much greater the difference in these angles would have been if the racing of the men had been *continued*, we are left to imagine, as it was convenient to cut short the experiment.

194. The deficiency of stability in the "Sultan" was further shown in that, while she took 60 oscillations after the men ceased to roll her before she had reduced her angle of

roll to 4°, the "Devastation," in 40 oscillations, had arrived at a roll of only seven-tenths of a degree!

195. It may be asked if the "Devastation's" shorter period has not produced rapid and injurious rolling—certainly not; for there is not much difference in the rapidity of roll, for while her period is shorter the arc she rolls through is only half! which involves less wear and tear, and allows greater facility for working her guns. Moreover, at her worst, she could point her guns three times nearly for the "Sultan's" twice, and after a time she would be at rest and able to fire with accuracy when the "Sultan" was still oscillating through large arcs.

196. Raise the centre of gravity of "Devastation," and she will become vastly more dangerous. Lower the centre of gravity of "Sultan," or in a new design, if it were reasonable to repeat such, and the result would be that she would steer better, sail better, roll less, pitch less, be safer, and fight her guns more easily and effectively—admit of having her bottom stronger to resist injuries from taking the ground or from torpedoes. In a word, there is everything to justify lowering her centre of gravity by "drawing from the dockyard" not a centre of gravity, but means of placing that of the "Sultan" where it ought to be, and ought to have been, but where it was never intended it should be.

197. But there is a test these ships should be put to, which involves conditions that may occur in their future history, if not disastrously cut short or otherwise prevented. Put them into the basin, and take out all coals, water, and provisions, powder and shot, except a limited amount, such as a ship would contain on arriving from foreign service or a long cruise, and then race 400 men across their decks the same as before, and note the effect as to stability—especially that of the "Sultan," as she is masted, and has such a limited amount.

198. It is impossible that a ship, upwards of 9,000 tons weight, could have been rolled through an angle of 29½° by moving 30 tons weight from side to side, as was the "Sultan,"

if she had had half the amount of stability that her alleged metacentric height assigns her.

199. This however is clear, that a small ship moving but with ordinary speed, and striking her high, *as she would*, would turn her over, especially if she struck when the "Sultan" was heeling away from her.

200. How, then, can a ship in that condition be fit to battle with winds and seas even now that she is deep? and how much more unfit when light of coals, stores, and provisions!

201. The "Devastation" is said to roll less and to be more stable than the "Sultan." But why,—except that she has a shorter period and greater stability?

202. This we said must be so, in opposition to Mr. Reed's statement in "Our Ironclads."

This view Professor Rankine confirmed, page 5, No. 26, Committee on Designs' Letter, March, 1871. Yet *Naval Science,* page 368, vol. i, denies this, and reaffirms the erroneous opinion, *i. e.*, that "the arc of roll of a ship has "sometimes been reduced by raising the common centre of "gravity."

203. Now we have the satisfaction of knowing that our recent ironclads were designed to have very little stability, upon the unfounded idea that this would increase the period of their roll and *would diminish* the arc through which they rolled, and obtain them "a steady platform."

204. The recent fact shows, on the contrary, that the "Devastation," with a period of 13·6 seconds and greater stability, rolls through only one-half the arc that the "Sultan" rolls through, with a period of 17·7 and less stability.

205. A consequence of all this ignorance is, that all these ships with small stability, even after the introduction of so much ballast, are liable, if run into even by a small ship, to be rolled over;—not over pleasant, after so much money has been spent to make them safe and useful!

206. When some of our ironclads have been run down, as the Italian ship in action, or the Spanish ship by accident, or turned over by being run into when lying in a roadstead,

or from the inability to steer them when getting under weigh, we shall understand that we have been wasting the resources of the country to make expensive coffins for the *personnel* of the once great Navy of England. These catastrophes are the more certain to occur to our ships because of their small stability, reduced on the plea of making them safe.

207. The fact is, overlooking that the immediate function of the Royal Navy is to preserve intact the rights of Englishmen, to keep the peace throughout the wide domain of England's commerce and colonies, and to combat slavery and piracy in every form, we have employed the resources of the country to turn our Navy into fighting machines which, if dangerous to our enemies, are far from being arks of safety to their crews.

ON STOWAGE OF SHIPS.

As the current mode of calculating ships' stabilities proceeds on the erroneous hypothesis, that it is a matter of indifference whether a ship possess a buoyant *life-belt* or a heavy danger-belt near her water-line, we will throw out some suggestions for the consideration of shipowners and stevedores.

Many legitimate causes have tended to the construction of narrow and deep ships, but such require more care in stowing.

Ships having an immersed depth equal to or greater than their half breadth, especially if flat-floored, have inherently little or no stability whatever. Such ships will be dangerous in proportion to their depth unless their centres of gravity in stowing are brought proportionably low. The spaces from near their sides, 2 feet to 6 or 7, according to the size of the vessel, above and below the load water-line in all ships, but especially the above class of ship, should be kept buoyant; if any weight is placed there it should be light; this is the *life-belt* of the ship.

If two ships of this design, of the same size and intended load immersion, were launched with their masts on end, and their yards across, as many Italian ships are, but one constructed of iron the other of wood, the iron ship would capsize if launched without ballast, the wood ship would not. The lighter bottom of the iron ship causes the centre of gravity of her hull to be higher than that of the wood ship.

If these two ships are to be sailed without cargo, the iron ship will require more ballast than the wood ship.

If both ships are loaded with light cargo, equally distributed, the iron ship will carry more, but would be in more danger because less stable; both probably would be better for some ballast, but the iron ship would require it, and more of it.

In both, care must be taken to preserve the *life-belt* complete.

In cases of light cargoes shipowners must not be afraid of a little ballast in the bottom; it is the most effective insurance, as it adds to the safety, and as it also makes the sails more effective, it tends to making a quicker passage, and limiting the arcs of roll it reduces the wear and tear of ship.

The amount of freeboard should be in proportion to the breadtb, for the broader the ship the sooner is the gunwale immersed at a given angle of inclination.

We suppose, then, the pressure of sail and stability to be equal in the two ships.

An empty bottom, loaded sides near the load-water line, as they destroy the *life-belt*, and a very low freeboard, are each dangers.

Associating all these dangerous elements in our design, particularly where the weights have been lifted 5 or 6 feet off the bottom, and a high centre of gravity given, indicates great ignorance of true principles; happily such is not often done.

Deep and narrow ships have been sometimes unduly disparaged on the plea that they are necessarily weak because of their great proportionate length, but this is to overlook that their great depth, as in the case of a girder, is a prime element of strength, and that a deep narrow ship is stronger than a wide shallow ship of the same content, length, and scantling; while the wider ship, all other things being equal, will be subject to greater wear and tear because of the increased action of the waves, due to their great area of load-water section.

Broad shallow ships have much inherent stability; if therefore much heavy weight be stowed in them low down, their stability will be so very unduly increased that their motions will be very quick, and these will endanger rolling the masts over the side.

Under such circumstances, a deck load (so far as the weight is concerned), if not too great, and if judiciously stowed, would be an advantage rather than a cause of danger.

Generally such ships, especially if paddle steamers, may carry a limited deck load without danger.

If the cargo should be of great specific gravity, it should be distributed vertically, not all lifted up off the bottom and concentrated above, a portion should be placed on the bottom, in which case some may with advantage be carried very high, even as a deck load, provided the *life-belt* is preserved.

There is no necessary danger in some high weights, even deck loads, any more than there is in the two or three decks of old vessels of war with their accompanying guns, if only the weight be centralized by a tumble home of the topsides, or by stowage, so only two things are attended to, the preservation of the *life-belt*, and the centre of gravity, if the system be in its proper position for sufficient stability.

If at the last moment comparatively heavy weights are crammed into a deep narrow ship high up over a light cargo, even though she is not immersed to her ordinary load water-line, she will be in danger, as her centre of gravity will be thus raised and her *life-belt* destroyed.

It should be observed, that fear of uneasy motions from having a low centre of gravity are least to be indulged in, in the case of deep and narrow ships, for the disturbing force being the waves, their effect will be in the proportion that the area of the load water-line section bears to the immersed body—and great depth is the great retarder of motion.

These positions proceed on the supposition that the ships have been endowed with a sufficient amount of stability.

If not, it may be asked, Who is responsible ?—the owner and stevedore may have done all that they were advised to do, and have done nothing to impair the stability according to the design—clearly the Naval Architect.

There can be no reasonable doubt but that many of the lives and ships lost are so, because the latter have not had sufficient stability, and this not because of their carrying deck loads.

Sometimes because the stevedore unwarned has not preserved the *life-belt*. Sometimes because the architect has not given enough margin of stability to allow for the deduction on account of the errors in the calculations.

The recommendations to reduce the stability of ships to a minimum that has been given currency to and practised in the Navy, and supported as it has been by "false facts and false theories," cannot be too strongly denounced, or the authors and abettors too distinctly held responsible,—the more necessary when the grave errors that must occur in their calculations are considered.

From the foregoing it may be perceived that as there is a divided responsibility, and that it is not so easy to legislate, with safety to the crews of ships or with justice to the shipowners, as some have imagined.

No doubt overloaded and unsound ships are very bad, and all aiders in sending such ships to sea should be held responsible, but so are such ill-conceived designs as those which eventuated in the loss of the "London," with a large body of passengers and crew, in no worse weather than an ordinary ship's boat, laden with a living cargo, could survive through.

And we know that merchant ships that put to sea without previous trial, built on the Froude-Reed system, designed to have little or *no* stability, for the latter is what it comes to by their errors, and with buoyant bottoms, and heavy topsides, would capsize, for such would have been the fate of such vessels built for the Navy, the "Invincible," "Sultan," "Audacious," "Iron Duke," and others, had not the earlier vessels been tried and found unseaworthy, and thence all largely ballasted.

We ask how are sailors' lives to be protected, if Naval Architects are allowed to play such antics, and then allowed to shelter themselves from all responsibility under pleas of science, ignorantly so called ?

The fact is, the formula as to the stability of ships was constructed on the supposition of a uniform distribution of weight and of buoyancy in the immersed body and in the volumes of immersion and emersion, while generally it is much otherwise.

Let Fig. XXX represent the main cross section of a ship, also all the others, as they differ only in degree, *c* represents the centre of buoyancy, *CE* the water-line when upright, *ab* that when inclined.

Now, when a ship is inclined, an additional part of one side, say that included by *daC*, is forced under water, and a part of the other side *dEb* is forced out, the practical effect on the buoyancy is as though a volume equal to *dEb* were moved over to *daC*, thereby increasing the buoyancy of the body on one side of the middle line *dB* over that of the other side by the double amount; but the effect of this is to form a new centre of buoyancy at say *c′*, the distance that *c* will be from *c′* will depend upon the proportion the volumes *daC* and *dEb* bear to the volume *CBE*, and upon their leverage, which varies with the breadth.

But this is on the supposition that the ship turns round the point *d*, which never is the case, and upon the further supposition that specific gravity of *daC* and *dEb* is precisely the same as that of the other part of the immersed body, which it never is.

If the specific gravity of *daC* and *dEb* is less than that of *CBE* then they will possess a greater proportionate buoyant power, and the centre of buoyancy will be moved by a given inclination a greater quantity, the distance from *c* to *c′* will be greater and the stability proportionably greater.

If the specific gravity of *daC* and *dEb* be made greater than that of *CBE*, then the distance from *c* to *c′* will be less, and the stability will be less also.

If the specific gravity of *daC* and *dEb* be made so great that they would sink if separated from the ship, they would not possess buoyancy, and therefore could not influence the centre of buoyancy, and their immersion and emersion could not add to the stability.

These volumes are what Mr. J. Scott Russell calls the shoulders upon which depends the stability or the power which tends to restore a ship to the upright when inclined or tends to resist inclination.

It must be obvious that if these shoulders do not possess buoyancy themselves, they cannot give any buoyant support or offer any buoyant resistance to inclination. And yet Naval Architects have been in the habit, in their calculations, of treating the question as to whether these solids possessed a very great deal of buoyancy or very little as a matter of indifference. The consequence of this explains the confession of the Admiralty Constructors, given at page 14.

Obviously then the specific gravity of the spaces daC and dEb should always be less than that of the main body CBE. —They constitute the *life-belt.*

Then, not to mend matters, came the Froude-Reed system, the most incorrect and the most dange ous, perhaps, that was ever conceived. Let Fig. XXXI represent one of these designs.

These gentlemen not simply recommend the centre of gravity to be kept high to reduce the stability, but the latter gentleman, to effect this more easily, added a deep empty space between an outer and an inner bottom; they also recommend a concentration of weight on or towards the sides, with a view to reduce the rolling, and Mr. Reed to reduce the arcs rolled through.

Mr. Reed testified that this latter was not the result, when he reported in the *Times* the great ironclads rolling more than the little wooden "Topaze," and a distinguished French Officer testified to the same effect. See page 47.

Now, the concentration of weight on the sides and above, from the armour, and for the purpose of raising the centre of gravity and for increasing the moments of inertia of the sides with the view of reducing the rolling, increases enormously the specific gravity of daC and dEb, so that they possess little buoyant power, and therefore their immersion or emersion exercises little influence on the motion of the centre of buoyancy, and therefore afford very little stability, the more particularly that the empty cells in the bottom give a cork-like buoyancy to it and increases the disproportion between the specific gravity of the main body *CBE* and the volume *daC* and *dEb*. If the tendency of this volume is to move *c* towards *c'*, the tendency of the bottom is to move the centre of buoyancy in the opposite direction, by each angle of inclination as the quantity of this cellular cork-like part on the right of the vertical *Bx* is increased. Moreover, the moment of inertia of the sides once overcome, this becomes another element to reduce the stability further and to increase the danger.

While the small moments of inertia of the lower parts facilitate the upward tendency of the bottom.

Hence, the dangerous blunders in the estimates of the stabilities of "Sultan," "Invincible," and others, which was only partially corrected by the ballast, which destroyed the buoyancy of the bottom; we anticipated, and stated so in the *Times*, 1870, the danger that since ensued.

Whatever may have been the defects of the "Captain," and whatever warning was given to her Captain or the First Lord of the Admiralty, no warning was given of those enormous diminutions of her stability. So she was lost, *secundum artem*, not that of the Sailor, but of the Naval Architect, who forced deep double bottoms and heavy top-sides into the Navy against the voice of warning.

The insurance of any vessel going to sea without an effective *life-belt* should be void, for though a vessel could be made safe from capsizing by a low centre of gravity, yet would she be a bad ship, steer badly, be crank and leewardly, and would finally be lost, herself and crew, on a lee shore.

NOTE.—We have represented in the two figures, XXXI and XXII, the water-line as moved, instead of the vessel, as that was the more general way in which "diagrams" are represented. We thought it would be best understood, not that we think it the more correct; on the contrary, we think it obscures some points.

APPENDIX A.

CENTRE OF PRESSURE.

The centre of pressure of a plane surface immersed in water, or sustaining water pressing against it, is that point, to which, if a force be applied equal and contrary to the whole pressure exerted by the water, the plane will remain at rest, having no tendency to incline to either side.

From this definition it follows, that if a plane surface immersed in water, or otherwise exposed to its influence, be parallel to the horizon; then, the centre of pressure and the centre of gravity occur in the same point, and the same is true with respect to every plane on which the pressure is uniform; but when the plane or surface on which the pressure is exerted, is anyhow inclined to the horizon, or to the surface of the water whose pressure it sustains; then, in order to find the centre of pressure, we have to solve the following problem.

Having given dimensions and position of a plane surface immersed in water, or otherwise exposed to its influence; it is required to determine the position of the centre of pressure, or that point, to which, if a force be applied equal and opposite to the pressure of the water, the plane shall remain in a state of quiescence, having no tendency to incline to either side.

Let *STN*, Fig. IX, be a cistern filled with water, and let *abch* be a rectangular plane immersed in it at a given angle of inclination to its surface; produce the sides *ha* and *cb* to meet the surface of the water in the points *e* and *f*; join *e* and *f*, draw *es* and *fr* perpendicular to *ef* which is in the surface of the water; draw *hs* and *cr* meeting *es* and *fr* at right angles in the points *s* and *r*; then is *hes* or *cfr*, the angle of the plane's inclination, and *hs*, *cr* are the perpendicular depths of the points *h* and *c*.

Let *P* be the position of the centre of pressure, and through *P* draw *Pm* and *Pn*, respectively perpendicular to *cb* and *ch* the sides of the rectangular plane; then *cb* and *ch* may be taken the axes of rectangular co-ordinates originating at *c*, and *Pm*, *Pn* are the corresponding co-ordinates, passing through *P* the centre of pressure, supposed to be situated in *P*.

Now, it is evident from the nature of fluid pressure, that the force of the water against the small area δ, is equal to the weight of a column of the fluid, whose base is the rectangular material point δ, and altitude the perpendicular depth of that point below the upper surface of the fluid. The area δ may be considered a rectangular or square point the side of which is equal to the breadth of a material line; hence δ is considered to be very small. Therefore, the force against δ, varies as $\delta \times hs$; but, $hs = eh \times \sin. hes$; and therefore the pressure on δ varies as $\delta \times eh \times \sin. hes$, and the effort or moment of this force, to turn the plane about the ordinate *Pm*, varies as $\delta \times eh \times \sin. hes \times Pn$, in which *Pn* is the length of the lever on which the force acts. But $Pn = eh - fm$ for $eh = fo$; hence by substitution, the force to turn the plane about the ordinate *Pm*, varies as

$$\delta \times eh^2 \times \sin. hes - \delta \times eh \times fm \times \sin. hes;$$

therefore, the accumulated effort of all the forces, to turn the plane round *Pm*, must be proportional to

$$\text{sum of } \{\delta \times eh^2\} \times \sin. hes - fm \times \text{sum of } \{d \times eh\} \times \sin. hes,$$

and this expression, by the definition, is equal to nothing;

$$\therefore fm = \frac{\text{sum of } \{\delta \times eh^2\}}{\text{sum of } \{\delta \times eh\}}.$$

The whole area *abch* contains an indefinite number of elementary areas δ; let δ' be any point on the plane, $pt = x$ and $qt = y$ the variable co-ordinates; then the area of δ' is represented by $dx \times dy$, that is the differential of x multiplied by the differential of y, written $dx\, dy$; d, being a symbol of operation, has no numerical value, no more than $\surd$ has in $\surd Q$ which is put to represent the square root of Q.

From conventional arrangements, the expression for fm becomes

$$fm = \frac{\int\int x^2 dx dy}{\int\int x dx dy} = \frac{\int x^2 y dx}{\int x y dx}; \qquad \text{(I.)}$$

$\int$, like d, is a symbol of operation, it stands for the sum of, and is termed the sign of integration. Of the operations indicated by these signs ($\int, \int\int, \int\int\int$; d, d^2, d^3), we will treat more fully hereafter.

By reasoning in the same manner as above, it will readily appear, that the effect or moment of the pressure on δ, to turn the plane about the ordinate Pn,

$$\text{varies as } \delta \times eh \times hn \times \sin. hes;$$

but, $hn = ch - cn$; therefore by substitution, the force on δ to turn the plane round Pn,

$$\text{varies as } \delta \times eh \times ch \times \sin. hes - \delta \times eh \times cn \times \sin. hes;$$

and consequently, the effect of all the forces to turn the plane around Pn, must be proportional to the

$$\text{sum of } \{\delta \times eh \times ch\} \sin. hes - cn \times \text{sum of } \{\delta \times eh\} \sin. hes;$$

but according to the definition, the sum of these forces is equal to nothing; for the plane has no tendency to incline to either side; therefore

$$Pm = cn = \frac{\text{sum of } \{\delta \times he \times ch\}}{\text{sum of } \{\delta \times he\}};$$

Hence, according to the conventional notation of the differential and integral calculus, we have,

$$Pm = \frac{\int\int xy dx dy}{\int\int x dx dy} = \frac{\int \frac{1}{2} y^2 x dx}{\int y x dx} = \frac{\int y^2 x dx}{2\int y x dx}; \qquad \text{(II.)}$$

It is evident that the inclination of the surface pressed to the surface of the fluid has no effect in altering the centre of pressure, except when the two are parallel, in which case the centre of pressure will evidently coincide with the centre of gravity.

If $y = k$, a constant quantity, then (I) becomes

$$fm = \frac{k\int x^2 dx}{k\int x dx} = \frac{\int x^2 dx}{\int x dx} = \frac{\frac{1}{3} x^3}{\frac{1}{2} x^2} = \frac{2}{3} x; \text{ and (II) becomes}$$

$Pm=\frac{k^2\int xdx}{2k\int xdx}=\frac{k}{2}$. Now if P_1 be the centre of pressure of the rectangle *efch*, $An=x$, and $ch=k$, then $AP_1=\frac{2}{3}x$ and $P_1m_1=\frac{k}{2}$.

Now if $AB=a$ and $An=b$, the integral $\int x^2dx$ between these limits becomes $\frac{1}{3}(b^3-a^3)$; and $\int xdx$ between the same limits is $\frac{1}{2}(b^2-a^2)$; consequently the distance of the centre of pressure of the rectangle *abch*, from the surface of the water *ef*, will be $\frac{2(b^3-a^3)}{3(b^2-a^2)}$; that is, if P be the centre of pressure of the rectangle *abch*, $AP=\frac{2(b^3-a^3)}{3(b^2-a^2)}$ and $Pm=\frac{k}{2}$.

$$\frac{2}{3}\left(\frac{b^3-a^3}{b^2-a^2}\right)=\frac{2}{3}\left(b+a-\frac{ab}{b+a}\right).$$

Required the position of the centre of pressure in the rectangle *abch* when AB = 30 inches, An = 78 inches, and hc = 42 inches.

$$\frac{2}{3}\left\{78+30-\frac{78\times30}{108}\right\}=57\tfrac{5}{9}\text{ inc.}=AP.$$

$$\therefore BP=57\tfrac{5}{9}-30=27\tfrac{5}{9}\text{ and }mP=21.$$

$$78-30=48=Bn\text{ and }48-27\tfrac{5}{9}=20\tfrac{4}{9}=Pn.$$

We will now show, if sh = 72 in., that a pressure of $3634\frac{8}{13}$ lbs. applied at the point P, in the direction of a perpendicular to the plane *abch*, will keep the plane at rest; this single pressure is equal to the whole pressure exerted by the water at the other side of the rectangular plane *abch*.

$$\frac{42\times48}{144}=14\text{ sq. ft. the area of } abch;$$

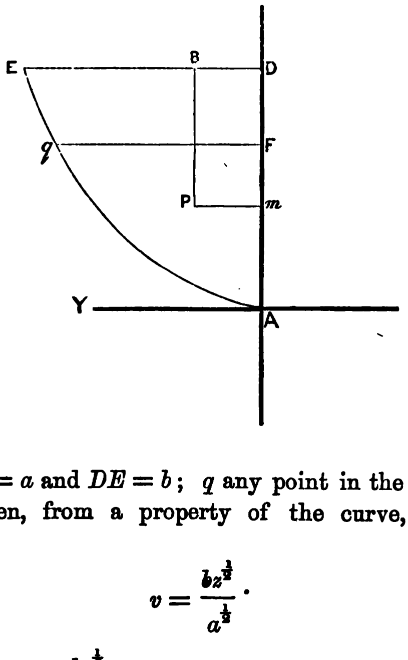
$sh=72$ in.=perpendicular depth of the water at h, and $eh=An$ $=78$ in. $\therefore\ \sqrt{78^2-72^2}=30=es$ since *esh* is a right-angled triangle.

$ea+$half $ah=54$ in., therefore, by similar triangles $78:72::54:\frac{648}{13}$ in., the perpendicular depth of the centre of gravity of the rectangle *abch* from the surface of the water

$\frac{648}{13}$ in. $= \frac{54}{13}$ feet; and since a cubic foot of water weighs about $62\frac{1}{2}$ lbs., the whole pressure exerted by the water on the rectangle $abch = \frac{54}{13} \times 14 \times \frac{125}{2} = 3634\frac{8}{13}$ lbs.

To find the centre of pressure P, of a semi-parabola ADE, Fig. X, the extreme ordinate DE coinciding with the surface of the water.

FIG. X.



Let $AD = a$ and $DE = b$; q any point in the curve, $AF = z$, $Fq = v$; then, from a property of the curve, $a : b^2 :: z : v^2$ and $\therefore$

$$v = \frac{bz^{\frac{1}{2}}}{a^{\frac{1}{2}}}.$$

Hence, if $a - z$ and $\frac{bz^{\frac{1}{2}}}{a^{\frac{1}{2}}}$ be respectively substituted for x and y in (I) we have,

$$BP = \frac{-\int (a-z)^2 \times \frac{bz^{\frac{1}{2}}}{a^{\frac{1}{2}}} dz}{-\int (a-z) \times \frac{bz^{\frac{1}{2}}}{a^{\frac{1}{2}}} dz} = \frac{-\frac{b}{a^{\frac{1}{2}}} \int (a-z)^2 \times z^{\frac{1}{2}} dz}{-\frac{b}{a^{\frac{1}{2}}} \int (a-z) \times z^{\frac{1}{2}} dz}$$

$$\frac{\int\{a^2 z^{\frac{1}{2}}dz-2az^{\frac{3}{2}}dz+z^{\frac{5}{2}}dz\}}{\int\{az^{\frac{1}{2}}-z^{\frac{3}{2}}\}dz};$$

For $d(a-z)=-dz=+dz$.

$$\therefore BP=\frac{\frac{2}{3}a^2z^{\frac{3}{2}}-\frac{4}{5}az^{\frac{5}{2}}+\frac{2}{7}:z^{\frac{7}{2}}}{\frac{2}{3}az^{\frac{3}{2}}-\frac{2}{5}z^{\frac{5}{2}}}$$; no correction is required,

for both numerator and denominator vanish when $z=0$.

$$\therefore BP=\frac{\frac{2}{3}a^2-\frac{4}{5}az+\frac{2}{7}z^2}{\frac{2}{3}a-\frac{2}{5}z.}.$$ When $z=a$ $BP=\frac{4a}{7}$.

Again, if $a-z$ and $\frac{bz^{\frac{1}{2}}}{a^{\frac{1}{2}}}$ be respectively substituted for x and y in (II), we have

$$Pm=\frac{-\int\frac{b^2}{a}z(a-z)dz}{-2\int\frac{b}{a^{\frac{1}{2}}}(a-)z^{\frac{1}{2}}dz}=\frac{b}{2a^{\frac{1}{2}}}\frac{\int(azdz-z^2dz)}{\int(az^{\frac{1}{2}}dz-z^{\frac{3}{2}}dz)}$$

$$=\frac{b}{2a^{\frac{1}{2}}}\frac{\frac{1}{2}az^2-\frac{1}{3}z^3}{\frac{2}{3}az^{\frac{3}{2}}-\frac{2}{5}z^{\frac{5}{2}}}$$

No correction is required as both the numerator and denominator vanish, as they should, when $z=0$.

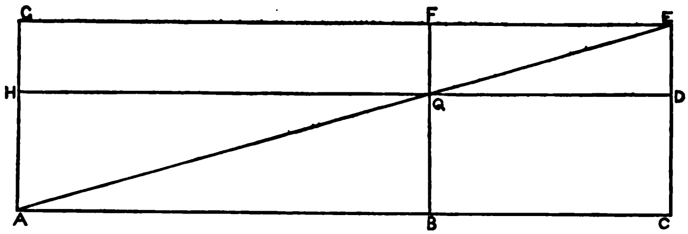
$$\therefore Pm=\frac{b}{2a^{\frac{1}{2}}}\frac{\frac{1}{2}az^{\frac{1}{2}}-\frac{1}{3}z^{\frac{3}{2}}}{\frac{2}{3}a-\frac{2}{5}z}$$; hence when $z=a$

$$Pm=\frac{5b}{16}.$$

GEOMETRICAL PROPOSITION.

It is proved, Euc. 43, I, that the complements of the parallelograms GQ, QC, which are about the diagonal AE of any parallelogram GC, are equal to one another.

Fig. XI.



Let GC, Fig. XI, be any rectangular parallelogram, $QD = b$, $DC = a$, and $GF = a + b$.

Then, if $x = QF$, we have

$$x\,(a + b) = ab; \;\therefore\; x = QF = ED = GH = \frac{ab}{a + b}.$$

In Fig. XII, let $ED = b$, $EA = EK = ER = a$, then if RF be made equal to $a + b$ and the diagonal FQG drawn, we shall have $DG = \dfrac{ab}{a + b}$.

Again, as $EK = AE$, $DK = b + a$; make $DH = DG$, then $HK = b + a - \dfrac{ab}{a + b} = \dfrac{b^3 - a^3}{b^2 - a^2}$. Draw HI perpendicular to KH and make $HL = \dfrac{1}{3}\,HI$; then $ML = \dfrac{2}{3}\,KH$ and $ML = \dfrac{2}{3}\left(\dfrac{b^3 - a^3}{b^2 - a^2}\right)$. The centre of pressure of the rectangular parallelogram $ABCD$ is formed by taking $OP = ML$.

TO FIND THE CENTRE OF PRESSURE OF A PLANE IMMERSED AS SHOWN IN FIG. IX.

Fig. XIIa shows the immersed plane; $EB = a$; $EL = b$; make $NL = a$, and complete the parallelogram $EFNL$. Draw ED perpendicular to LE, and equal to $2\ a + b$, that is $FD = a + b$. Draw DNM cutting EL produced in the point M; make $EH = a$, and $LT = LM$, draw TS perpendicular to LH; in TS take any point K, and make $KS =$ twice KT; draw KR parallel to MH. Then $RK = GP_2 = JP_3$, the distance of the centre of pressure of the point P_3 from the surface of the water. Let $LM = x$, then (see Fig. XI) $x \times (a + b) = LM \times FD = EL \times EF = b \times a$; $\therefore x = \dfrac{ab}{a+b} = LM$ and $LH = b + a \therefore TH = = a + b - \dfrac{ab}{a+b} = \dfrac{a^2 - b^2}{a^2 - b^2}$.

DETERMINATION OF THE EFFECTS OF AN UNEQUAL DISTRIBUTION OF WEIGHT ON THE SUPPOSITION THAT THE CENTRE OF GRAVITY IS THE AXIS OF ROTATION.

Taking the data given in Articles 86 to 109, we will in the next place critically examine the nature and circumstances attending the arrangement exhibited in Fig. IV.

LIKM, Fig. XIII, is a cross section of a parallelopiped of given length; this section is taken in the middle of the body, equidistant from the ends. $IK = KM = ML = LI = 12$ ft. The reasoning for one foot of length taken in the middle of the body will hold good for any given length; so we have taken the length $= 1$ foot; through the centre of which the section KL is supposed to pass. The water line Nr passes through the centre of gravity g, of the body cutting the sides LI, KM, at t and r; rM is taken $= 9$ feet and $tL = 3$ feet.

$rP = \dfrac{2}{3} rM$, see page 66, therefore, P, is the centre of pressure of the water on rM, pointed out by the arrow at P. $tQ = \dfrac{2}{3} tL$, therefore, Q is the centre of pressure of the water on tL, pointed out by the arrow at Q. It should be here remarked that a in the formula, page 66, is taken $= o$, as r and t are in the water-line. The position assumed makes $NM = 2\ Mr$, $NL = 2\ Lt$, hence $NL = 6$ feet $= MT = Tg$. Take $NA = 2\,NL + NM$, draw ATD, and make $Mm{=}MD$; mG may be taken a perpendicular of any convenient length, then take $GE = 2mG$; GF, parallel to $BM,{=}NO$, then O is the centre of pressure of the water on LM, pointed out by the arrow at O. NB is taken equal to NL. By calculation it will be found that $NO{=}13$ ft.

Now we may suppose the water to be removed if the pressures at O, P, and Q be retained. The pressure Q acts upon the lever $gf = 5$ ft., and tends to turn the body round its centre of gravity g. Let $20s = 1000$, the specific gravity of water.

$Nt = 3\sqrt{5}$, $Le = et$; $3\sqrt{5} : 6 :: 3 : \frac{6}{\sqrt{5}}$, hence $\frac{3}{\sqrt{5}}$ = the perpendicular distance of e from the surface of the water; therefore, $\frac{3}{\sqrt{5}} \times (3 \text{ sq. ft.}) \times 20s = \frac{180}{\sqrt{5}}\, s$ = the pressure of the water on the three square feet of surface between t and L, in terms of s; this pressure can be reduced, to a weight in pounds, if required.

Therefore, the moment of Q, tending to turn the body round g, may be represented by $\frac{180}{\sqrt{5}}\, s \times (5 \text{ ft.}) = \frac{900}{\sqrt{5}}\, s$.

The perpendicular distance, of the centre of the side LM, from the water-line $= \frac{12}{\sqrt{5}}$, for, $6\sqrt{5} = Ng : 12 :: 6 : \frac{12}{\sqrt{5}}$.

$\therefore \frac{12}{\sqrt{5}} \times (12 \text{ sq. ft.}) \times 20s$ = the pressure of the water on the twelve square feet of water between L and M, in terms of s.

Hence, the moment of O, tending to turn the body round g, may be represented by $\frac{12}{\sqrt{5}} \times (12) \times 20s \times g, 4$; but the lever g, 4, upon which the force acts $= 1$ ft., since $NO = 13$ ft.

$\therefore$ the moment of $O = \frac{12}{\sqrt{5}} \times 12 \times 20s \times 1 = \frac{2880}{\sqrt{5}}\, s$, in terms of s.

The pressure P acts upon the lever g, 7, $= 3$ ft., and tends to turn the body in an opposite direction round the centre of gravity g.

$$Mq = qr = 4{\cdot}5 \text{ ft.}$$

$Nr = 9\sqrt{5}$ and $9\sqrt{5} : 18 :: 9 : \frac{18}{\sqrt{5}}$ half of which $= \frac{9}{\sqrt{5}}$.

$\frac{9}{\sqrt{5}} =$ the perpendicular distance of q from the water-line Nr.

$\therefore \frac{9}{\sqrt{5}} \times (9 \text{ sq. ft.}) \times 20s = \frac{1620}{\sqrt{5}}s =$ in terms of s, the pressure of the water on the 9 sq. ft. of surface between M and r. Therefore, the moment of P, tending to turn the body round g, may be represented by $\frac{1620}{\sqrt{5}}s \times (3 \text{ ft.}) = \frac{4860}{\sqrt{5}}s$.

Since $\frac{4860}{\sqrt{5}}s - \frac{2880}{\sqrt{5}}s - \frac{900}{\sqrt{5}}s = \frac{1080}{\sqrt{5}}s$, the moment, in terms of s, tending to turn the body round g, in the direction indicated by the bent arrow, Fig. XIII, $= \frac{1080}{\sqrt{5}}s$.

It is evident that the moment of w_1 acting at its centre of gravity 1, will balance the moment of w_6 acting at its centre of gravity 6; the same may be said of w_2 and w_5, and of w_3 and w_4. Hence $\frac{1080}{\sqrt{5}}s =$ the moment, in terms of s, tending to turn the body LK round its centre of gravity g. The weight represented by s is about $3\frac{1}{8}$lbs. If a centre of pressure P, on the outside of a large vessel, be taken one or two inches above or below its true position, the error, practically speaking, will not much affect the value of the resultant; this, as the sequel will show, is a very important circumstance in favour of the system proposed by the writer, whether it be considered in a statical or in a dynamical point of view.

To find the centre of gravity of the trapezium $LMrt$, Fig. XIV. The centre of gravity is evidently in the line eq which is drawn from the middle of tL to that of rM; and it is easily shewn that the point c, in the line eq, whose perpendicular distance from the line $Mr = 5$ ft. is the centre of gravity of the trapezium $tLMr$. OM is the perpendicular distance of c, from the line Mr.

Let $LM = m$, $Mr = b$ and $Lt = a$. Draw tP parallel to LM;

LMPt is a rectangle and *Ptr* is a right-angled triangle. The area of $LP = ma$, and the area of the triangle $tPr = \frac{m}{2}(b-a)$; the distance of the centre of gravity of *LP* from $Mr = \frac{m}{2}$; and that of the triangle from the same line $= \frac{1}{3}m$.

Puting v for the distance of the centre of gravity of the whole trapezium *tLMr* from *Mr*, we have,

$$v\left(ma + \frac{m(b-a)}{2}\right) = ma \times \frac{m}{2} + \frac{m}{2}(b-a) \times \frac{1}{3}m.$$

$$\therefore v\left(a + \frac{b-a}{2}\right) = \frac{ma}{2} + \frac{b-a}{2} \times \frac{m}{3}.$$

$$\therefore v\left(\frac{a+b}{2}\right) = m\left(\frac{a}{2} + \frac{b-a}{6}\right),$$

$$\text{and } \therefore v(a+b) = \frac{m}{3}(2a+b);\ \text{hence, } v = \frac{m}{3}\left(\frac{2a+b}{a+b}\right).$$

$$\text{Consequently } OM = v = \frac{12}{3}\left(\frac{6+9}{3+9}\right).$$

Then, we have $NO = 13$ ft., the half of which, $Op = 6\frac{1}{2}$ ft.; $Nf = 12$ ft., $fg = 6$ ft., therefore, $Np = 6\frac{1}{2}\sqrt{5}$ and $Ng = 6\sqrt{5}$. The triangles *NOp*, p_zO being similar, we have

$$pN : NO :: pO : Oz = 2ch;\ \text{and}$$

$$Np : pO :: pO : pz = 2ph.$$

$$\text{Hence, } 6\tfrac{1}{2}\sqrt{5} : 13 :: 6\tfrac{1}{2} : \frac{13}{\sqrt{5}} = Oz;\ \text{therefore, } ch = \frac{6\frac{1}{2}}{\sqrt{5}}.$$

$$\text{Again, } 6\tfrac{1}{2}\sqrt{5} : 6\tfrac{1}{2} :: 6\tfrac{1}{2} : \frac{6\frac{1}{2}}{\sqrt{5}} = pz;\ \text{therefore, } ph = \frac{3\frac{1}{4}}{\sqrt{5}}.$$

$$\text{Now, } Np - ph = Nh,\ \text{that is, } 6\tfrac{1}{2}\sqrt{5} - \frac{3\frac{1}{4}}{\sqrt{5}} = Nh;$$

$$\text{but, } 6\tfrac{1}{2}\sqrt{5} - \frac{3\frac{1}{4}}{\sqrt{5}} = \frac{6\frac{1}{2} \times 5 - 3\frac{1}{4}}{\sqrt{5}} = \frac{29\frac{1}{4}}{\sqrt{5}} = Nh;$$

$$\text{and } Ng = 6\sqrt{5} = \frac{6 \times 5}{\sqrt{5}} = \frac{30}{\sqrt{5}}.$$

Consequently *Ng* is a little longer than *Nh*; therefore the difference, $gh = \frac{\frac{3}{4}}{\sqrt{5}} = \frac{3}{4\sqrt{5}} = \cdot335410196$ feet, or 3 inches nearly.

gh is technically termed *the equilibrating lever.* It is a straight line equal to the horizontal distance between the verticals, passing through the *centre of gravity* and the *centre of buoyancy;* or it is the horizontal distance between *the line of pressure* and *the line of support.* There are several terms, of very frequent occurrence, in the doctrine of flotation, which we will hereafter explain, before we proceed to develop the practical and true laws that regulate the conditions of stability. At present, we only direct attention to gh, *the equilibrating lever*, the length and position of which has never, in one single instance, been determined with sufficient accuracy to be of any practical value to Naval Architects, although they have made millions upon millions of calculations and experiments to find the position and length of this line, which is very short in all cases. Through the whole range of applied mathematics, the writer is only acquainted with a single hopeless parallel case, namely, that of measuring the parallax of the fixed stars, in order to find their distance from the earth. Mathematicians and writers on Naval Architecture have been led astray by trying to utilize the abstract truth, that the weight of the water displaced multiplied by gh gives the same turning power, or moment, as the combined pressures applied at O, P, and Q. In the case before us, where the length of every line can be found with strict mathematical accuracy, we see that the weight of

$tLMr$ multiplied by gh = (72 cubic feet of water) $\times \frac{3}{4\sqrt{5}} =$

$$72 \times 20s \times \frac{3}{4\sqrt{5}} = \frac{1080}{\sqrt{5}}s.$$

But it is illusory and dangerous, in the highest degree, to suppose that a correct value of the *equilibrating lever* can be found for large compound vessels or ships.

In a 3000-ton ship, an error, in the length of gh, of the 100th part of a foot (which is less than $\frac{1}{8}$th of an inch), would give an error in the varying resulting moment of 67200 foot-pounds for

$$3000 \times 2240 \times \frac{1}{100} = 67200 \text{ foot-pounds.}$$

It is no assumption that an error of $\frac{1}{8}$th of an inch *may* occur, since it must always occur in estimating the stability by an equilibrating lever which varies at each angle of increased inclination.

Hence, dealing with the outside varying pressures is practical and safe, especially when motion ensues; while all endeavours to utilize the varying pressures on the equilibrating lever *gh*, are illusory and dangerous.

These comparisons are made supposing the body to be at rest and in smooth water, and that the solid mass *tLMr* is allowed to do the business of the water which it has displaced; this state of things never does, and never can, happen at sea. When the body is in motion, and turning round an axis, we have to consider the *moments of inertia* of the parts of which the floating body is composed.

To find the moments of inertia of rectangular parallelopipeds w_1, w_2, w_3, *&c., moving round an axis passing through g perpendicular to the plane KL.* Fig. XIII.

Let *RTKP*, Fig. XV, be a rectangular parallelopiped,

$SN = a$, $SR = b$, $ST = c$, $SO = ON = \frac{a}{2}$. In the face *TN*

draw *OZe* perpendicular to *SP* and make *OZ* the axis of *z*. Suppose Z_2Z_3 to be the axis around which the body *TP* moves; then, Z_2Z_3 is parallel to *OZ*. *OB* is supposed to be perpendicular to *ON* and also to *OZ*.

Let $OA =$ the variable z; draw *AQ* parallel to *OB*, then $AQ = OB$, which put $= n$, a constant distance. It is evident that *BO* is equal to the variable z and that *AOBQ* represents a rectangular parallelogram. *OX*, that is *OB* produced, is taken for the axis of the variable x; *ON*, one of the edges of the parallelopiped, perpendicular *XOB*, is taken for the axis of the variable y. The position of the very small parallelopiped δ, whose conterminous sides are represented by dx, dy, dz, is indicated by the variable rectangular co-ordinates x, y, z.

$OD = x$, $DE = y$, and from *E* to the material point δ equals z.

Now, the *moment of inertia* of a system, in respect to an axis, is the sum of the products which arise from multiplying each element, $dx\ dy\ dz$, of the system by the square of its distance from the axis. The moment of inertia is always a positive quantity, and always increases when the mass of the system increases.

The general solution of this problem plainly consists in expressing, analytically, by means of the co-ordinates, x, y, z, the product of an element, $dx\, dy\, dz$, multiplied by the square of its distance from the given axis $Z_2\, Z_3$, and in afterwards integrating for the whole extent of the system. Q, C, δ, is a right-angled triangle, right-angled at C; $QC = n + x$, therefore $(n + x)^2 + y^2 =$ the square of the distance of the point dx, dy, dz, from the axis Z_2, Z_3. The moment of inertia for the single element will, therefore, be

$$n^2 + 2n\, x + x^2 + y^2)\, dx\, dy\, dz.$$

But to find the moment of inertia of the immense number of elements of which the body is composed, we have to integrate successively, in relation to each of the three variable quantities x, y, z.

$$\int\int\int \{n^2 dx dy dz + 2nx dx dy dz + x^2 dx dy dz + y^2 dy dx dz\};$$

when integrated with respect to z, becomes

$$\int\int \{n^2 z dx dy + 2nzx dx dy + zx^2 dx dy + zy^2 dy dx\};$$

which, when integrated with respect to x, becomes

$$\int \left\{ n^2 xz dy + nx^2 dy z + \frac{1}{3} x^3 z dy + xzy^2 dy \right\},$$

lastly integrating with respect to the variable y, we have

$$xyz \left\{ n^2 + nx + \frac{1}{3} x^2 + \frac{1}{3} y^2 \right\};$$

no correction is required for the whole vanishes, when $x = o$, $y = o$, $z = o$. But when $x = b$; $y = \frac{a}{2}$; and $z = c$ the ultimate integral becomes $b\left(\frac{a}{2}\right)c\left\{ n^2 + nb + \frac{b^2}{3} + \frac{a^2}{12} \right\}$, which is the moment of inertia for half the body, $HPKI$, the double of which is

$$abc \left\{ n^2 + nb + \frac{b^2}{3} + \frac{a^2}{12} \right\},$$

the moment of inertia of the whole parallelopiped RK. Putting ρ for the density, which is supposed to be constant throughout the solid, we have $\rho abc \left\{ \left(n + \frac{b}{2}\right)^2 + \frac{a^2 + b^2}{12} \right\}$

for the moment of inertia for any particular uniform density ρ. In the case under examination $\rho = 24s$ in w_1 and w_6; $\rho = 5s$ in w_2 and in w_5; and $\rho = s$ in w_3 and w_4.

Moment of inertia of

$$w_1 = 24s \times (24 \text{ cubic feet}) \times \left(5^2 + \frac{12^2 + 2^2}{12}\right) = 21504s;$$

the moment of inertia of w_6 is also $= 21504s$ in terms of s.

Moment of inertia of

$$w_2 = 5s \times (24 \text{ cubic feet}) \times \left(3^2 + \frac{12^2 + 2^2}{12}\right) = 2560s;$$

$2560s$ is also the moment of inertia of w_5.

Moment of inertia of

$$w_3 = s \times (24 \text{ cubic feet}) \times \left(1^2 + \frac{12^2 + 2^2}{12}\right) = 320s;$$

this is also the moment of inertia of w_4.

Therefore, since the moment of inertia is always positive, $2\,(21504s + 2560s + 320s) = 48768s$ represents the moment of inertia of w_1, w_2, w_3, w_4, w_5, w_6 in terms of s, according to the arrangement shown in Fig. IV*a*. (Plate XVI.)

To find by construction the centre of pressure on the side *LM*, Fig. V*a*.

Make $AE = 6$ feet; $AD = 12$ feet; Fig. XII, $ER = 6$ feet $= DQ$; $RF = 18$ feet; $DG = 4\frac{1}{2}$ feet; $HK = 24 - 4\frac{1}{2} = 19\frac{1}{2}$ feet; $LM = \frac{2}{3}$ of $HK = \frac{2}{3}$ of $19\frac{1}{2} = 13$ feet $= OP$; P being the centre of pressure.

We shall in the next place, compare these results with those of a similar nature, obtainable from the arrangement of w_1, w_2, w_3, w_4, w_5, w_6, shewn in Fig. V.

Referring to Fig. V*a*, and supposing the compound body *KL* to be moved round an axis through g, by some external force, from the position shewn in Fig. I, through the same angle as that indicated in Fig. IV*a*, namely, 26° 34″; $Mr = 6$ ft., and $Lt = 3$ ft., in both cases. The pressure of the water at O, P, Q, Fig. V*a*, is not altered, it is the same as in Fig. IV*a*; but the moments of inertia of w_1, w_2, w_3, w_4, w_5, w_6, in Fig. V*a*, differ much from the moments of inertia of the same bodies arranged in the manner shewn in Fig. IV*a*. With respect to the position of the bodies and the order represented in Fig. V*a*, we have, the moment of inertia of

$$w_3 = s \times (24 \text{ cubic feet}) \times (5^2 + \left(\frac{12^2 + 2^2}{12}\right) = 896s.$$

$896s$, also represents the moment of inertia of w_4, which is supposed to move round the same axis through g, the centre of gravity of the compound body *KL*, Fig. V*a*.

The moment of inertia of

$$w_2 = 5s \times (24 \text{ cubic feet}) \times \left(3^2 + \frac{12^2 + 2^2}{12}\right) = 2560s.$$

2560*s* is also the moment of inertia of w_5. The positions of the two bodies w_2 and w_5, have not been altered and might have been left out of the comparison of the two arrangements shewn in Figs. IV*a* and V*a*.

The moment of inertia of

$$w_1 = 24s \times (24 \text{ cubic feet}) \times \left(1^2 + \frac{12^2 + 2^2}{12}\right) = 7680s.$$

7680*s*, represents in terms of *s*, the moment of inertia of w_6, moving round the same fixed axis through *g*.

Therefore, since the moments of inertia, of bodies rigidly connected and supposed to move round the same axis, are always positive, we have,

$2 \times (896s + 2560s + 7680s) = 22272s =$ in terms of *s*, the moment of inertia of w_1, w_2, w_3, w_4, w_5, w_6, arranged in the manner shewn in Fig. V*a*. Now the moment of inertia Fig. IV*a* $= 48768s$, which is more than double that of Fig. V*a*; hence, the body, arranged as in Fig. IV*a*, will turn and oscillate, from the application of unbalanced forces, more than the body arranged as in Fig. V*a*. The body *KL*, Fig. IV*a*, after turning and oscillating from side to side, will come to a state of rest in the position, Fig. IV*c*, the disturbing causes being removed, the pressure of the water at *Q* and *R* only being in action.

The forces *R* and *Q*, in Fig. IV*c*, being equal and opposite have no tendency to turn the body *LK*.

The compound body, Fig. IV*a*, will not come to a state of repose in the position shewn in Fig. IV*b*, although the disturbing forces be removed and the pressure of the water at *O* and *P* have no power to turn the body one way or another, for, the centre of gravity *e*, of the compound triangular body, compound because of the unequal distribution of the weight. *LKM* (not the centre of gravity of displacement), is $\frac{217}{90}\sqrt{2}$ feet from the water-line *KL*. $\frac{217}{90}\sqrt{2} = 3{\cdot}409826$ ft., while, the centre of gravity e_1, of the compound parallelopiped *tK* is $4\frac{8}{15}$ feet from the water-line *tr*.

The position in which the immersed portion has its centre of gravity lowest, is the position in which the body will permanently rest. But, as the moment of inertia of the body, Fig. IV*a*, is great, it is not difficult to make *LI* take the place of *MK*, Fig. IV*c*.

The body *IM*, Fig. V*a*, its moment of inertia being small, comparatively speaking, will oscillate from side to side and come to a state of rest in the upright position shewn in Fig. V*c*, the disturbing causes being removed and the pressure of the water at *Q* and *R* being only allowed to act; the forces *Q* and *R*, Fig. V*c*, being equal and opposite have no tendency to turn the body *IM*.

The compound body V*a*, will not come to a state of rest in the position shewn in Fig. V*b*, although the disturbing forces be removed, and the pressure of the water at *O* and *P* have no power to turn the body one way or another, for, the centre of gravity e_2 (not the centre of gravity of displacement) of the compound triangular body *LMK*, is $\frac{74}{45}\sqrt{2} = 2.3255956$ ft., from the water line *LK*, while the centre of e_3, of the compound parallelopiped *tM* is 3 feet from the water line *tr*. Hence the body Fig. V*a*, will come permanently to rest in the position Fig. V*c*.

The moment of inertia of the body V*a* being small, comparatively speaking, it may be made to oscillate freely from side to side, but it would be difficult to make *IK* take the place of *LM*, Fig. V*c*, by any moderate force of wind or wave. Before proceeding further with respect to the motions and oscillations of bodies floating in water, it is necessary to solve a problem which mathematicians and writers on mechanics have left in a very unsettled state; this problem is:—To find the principal axis of rotation of a compound body, like a ship at sea, when acted upon by external forces applied at different points and in directions that vary.

OF ROTATORY MOTIONS AND THE PRINCIPAL AXES OF ROTATION OF A SOLID BODY.

When a body receives, at the same instant, impulses which are separately able to produce rotation about different known axes, the result will be, that the body will rotate about a new and determinable axis. The problem to be solved may be simplified and stated thus:—Let a body tend to turn at the same instant about each of the three rectangular axes OX, OY, OZ, with the respective angular velocities v_1, v_2, v_3; the portion of the axis about which the body will actually turn and the angular velocity of the rotation are required.

Let δ_1 be a particle of the body, $OB = x$; $BC = y$; $C\delta_1 = z$. δ_1 is a very small parallelopiped whose conterminous sides dx; dy; dz; are supposed to be parallel to $OB = x$; $OA = y$; $OE = z$, respectively. The very short line called the differential of x, and conventionally represented by dx, not to be taken as the product of d and x, is supposed to be always of the same length for every position of the particle δ_1, when x is taken as the independent variable. But dy and dz may be equal, greater, or less than dx, according as the lengths and positions of y and z vary. The plane of the circle $F_1\delta_1G_1H_1$, perpendicular to OY, is parallel to the plane of the circle $F\delta GH$, which passes through the axes OX and OZ. $\delta_1\delta$ is supposed to be drawn perpendicular to plane of $H_1G_1\delta_1F_1$ and consequently perpendicular to the plane $HG\delta F$.

So far, the ideas that Fig. XVII assists to convey being clearly understood, let us first consider the motion of δ_1 about the axis OY; this will be in a circle $H_1G_1\delta_1$, and we shall suppose it to be in the direction of the arrow. Suppose δ_1 to be transferred to δ, the plane of the circle $HG\delta$ being parallel to the plane $H_1G_1\delta_1$, the particle at δ_1, or at δ has no motion in the direction of OY; and the motion of δ in the directions of OX, OZ, will obviously be the same as if the circle $H_1G_1\delta_1$ coincided with $HG\delta$. The body of which δ_1 is a particle may be of any known form whatever, and all the innumerable small parallelopipeds, like δ_1, of which we suppose the body to be composed, may be imagined to be transferred in a similar way to the plane of ZOX while we examine the turning motion of the body about the axis OY.

As δ_1 or δ turns towards the plane XOY, its co-ordinate x increases, and its co-ordinate z diminishes; therefore, the differential of x, (dx), with respect to the time, will be positive, and that of z, or dz, negative. Now observe, x is no longer the principal variable, but changes with the position of δ_1 like dy, dz; the principal variable is the time t, which may be taken in seconds, and the differential of t, written dt, is the shortest time that can possibly be imagined. The absolute velocity of δ about OY will at any time t seconds (t''), corresponding to the co-ordinates x, y, will be in the direction of the tangent δr, taking, therefore, this line to represent it, its components in the directions OX, OZ, will be δq, and δp. Since the expression for the absolute velocity is $O\delta \times v_2 = \delta r$; the velocities v_1, v_2, v_3, are taken at a unit of distance from O. Since $\delta q = \delta r \times \sin. \delta r q$, and the angle $qr\delta = \delta OB$; therefore, $\delta q = \delta r \times$ sine of $\delta OB = v_2 \times O\delta \times \sin. \delta OB = v_2 \times \delta B = v_2 z$. Therefore, $v_2 z = \delta q$, expresses the velocity in the direction of OX which is due to the rotation about OY. Again, $\delta p = \delta r \times \cos. r\delta p = O\delta \times v_2 \times \cos. r\delta p = v_2 \times O\delta \times \cos. \delta OB = v_2 x$, which represents the velocity in the direction OZ to be taken negatively; therefore, the velocities in the directions OX, OZ, due to the rotation of any particle δ_1, at the point x, y, z at any time t seconds (t'')

$$\text{are } v_2 z \text{ and } - v_2 x.$$

In the next place let us consider the rotation of the particle δ_1 about the axis OX, from Y towards Z; suppose δ_1 to be transferred to D in the plane YOZ, and applying a similar process of reasoning we shall find for the velocities in the directions OZ, and OY, the values

$$v_1 y \text{ and } - v_1 z.$$

Finally, a similar process of reasoning may be applied when the rotation is about OZ from OX towards OY; suppose δ_1 to be transferred to C in the plane XOY, we have, for the velocities in the directions OY, OX,

$$v_3 x \text{ and } - v_3 y.$$

When all these rotations have place simultaneously we have the partial velocities $v_2 z$ and $- v_3 y$ along the axis OX; $v_3 x$ and $-v_1 z$ along the axis OY; and $v_1 y$ and $-v_2 x$ along the axis OZ. During the time dt the distances described along the axis OX, OY, OZ, may be represented by dx, dy, dz respectively; hence, the following expressions for the whole velocities in these directions will be,

$$\left.\begin{aligned} \frac{dx}{dt} &= v_2 z - v_3 y \\ \frac{dy}{dt} &= v_3 x - v_1 z \\ \frac{dz}{dt} &= v_1 y - v_2 x \end{aligned}\right\} \qquad (A).$$

From these expressions, we may find about what axis the body actually turns at the instant t seconds (t''), for when the motions have place simultaneously every particle in the axis of instantaneous rotation is motionless; consequently,

$$\left.\begin{aligned} \frac{dx}{dt} &= 0; \text{ that is, } v_2 z - v_3 y = 0; \\ \frac{dy}{dt} &= 0; \text{ that is, } v_3 x - v_1 z = 0; \\ \frac{dz}{dt} &= 0; \text{ that is, } v_1 y = v_2 x = 0; \end{aligned}\right\} \qquad (B).$$

Because v_1, v_2, v_3 are given numbers, these three equations are of a straight line in space passing through the origin O of the axis; two of these equations are sufficient to determine the position of this line; the equations $x = \frac{v_1}{v_3} z$; and $y = \frac{v_2}{v_3} z$; may be taken. In these equations $\frac{v_1}{v_3}$ and $\frac{v_2}{v_3}$ are the trigonometrical tangents of the angles which the projections of the instantaneous axis make with the axis of z. If we take any point in the instantaneous axis, whose co-ordinates are x_1, y_1, z_1, then the distance of that point from the origin O, will be equal

$$(x_1^2 + y_1^2 + z_1^2)^{\frac{1}{2}} = \left(\frac{v_1^2}{v_3^2} z_1^2 + \frac{v_2^2}{v_3^2} z_1^2 + z_1^2\right)^{\frac{1}{2}} = \frac{z_1}{v_3}(v_1^2 + v_2^2 + v_3^2)^{\frac{1}{2}};$$

hence, if α, β, γ, denote the angles which the instantaneous axis of rotation makes with the axes OX, OY, OZ, we have

$$\cos\alpha = \frac{v_1}{v_3} z_1 \text{ divided by } \frac{z_1}{v_3}(v_1^2 + v_2^2 + v_3^2)^{\frac{1}{2}} = \frac{v_1}{(v_1^2 + v_2^2 + v_3^2)^{\frac{1}{2}}};$$

and $\cos\beta = \dfrac{v_2}{(v_1^2 + v_2^2 + v_3^2)^{\frac{1}{2}}}$; $\quad \cos\gamma = \dfrac{v_3}{(v_1^2 + v_2^2 + v_3^2)^{\frac{1}{2}}}$;

and hence, the position of the required axis may be found in terms of the known velocities v_1, v_2, v_3.

To find the angular velocity of the body about the instantaneous axis we have to consider only the angular velocity of any single particle taken at any point we please; let the particle be taken on the axis of x; from the particle whose distance from O along the axis $OX = x$, draw a perpendicular p to the instantaneous axis; then

$$p = x \sin a = x (1 - \cos^2 a)^{\frac{1}{2}} = \frac{x (v_2^2 + v_3^2)^{\frac{1}{2}}}{(v_1^2 + v_2^2 + v_3^2)^{\frac{1}{2}}}.$$

Put V for the absolute velocity of the particle whose perpendicular distance from the axis $OX = p$; it must be observed that p is perpendicular to the instantaneous axis and not to OX. Then we have

$$V + \left(\frac{dx^2 + dy^2 + dz^2}{d_\backslash + {}^2}\right)^{\frac{1}{2}} \text{ which becomes } x (v_2^2 + v_3^2)^{\frac{1}{2}}$$

for the position of the particle is so selected that in the second members of equations (A), $y = 0$ and $z = 0$. Then if v be put for the angular velocity, that is the velocity at the distance of a unit from the instantaneous axis of rotation, we have

$$v \text{ the angular velocity} = \frac{V}{p} = (v_1^2 + v_2^2 + v_3^2)^{\frac{1}{2}}; \quad (C).$$

We have now established the following abstract truths, which are capable of being practically applied, namely, that three simultaneous angular velocities v_1, v_2 v_3, about three rectangular axes OX, OY, OZ, are equivalent to the single angular velocity $(v_1^2 + v_2^2 + v_3^2)^{\frac{1}{2}}$ about an axis inclined to OX, OY, OZ at angles whose cosines are respectively,

$$\cos a = \frac{v_1}{(v_1^2 + v_2^2 + v_3^2)^{\frac{1}{2}}}; \quad \cos \beta = \frac{v^2}{(v_1^2 + v_2^2 + v_3^2)};$$

$$\cos \gamma = \frac{v_3}{(v_1^2 + v_2^2 + v_3^2)^{\frac{1}{2}}}.$$

And that, when a body revolves about any axis given in position, and with a given angular velocity v, we can always resolve this motion, as far as velocity is concerned, into three partial rotary motions about three rectangular axes of co-ordinates posited as indicated by Fig. XVII. For the equations of the axis of rotation on the planes ZOX, ZOY, YOX, compared with equations (B), furnish two independent equations among the unknown quantities v_1, v_2, v_3; then, by calling in the assistance of the independent equation (C) we are able to find the values of v_1, v_2, v_3. In equations (B), this considered, x, y, z, become known quantities.

PRINCIPAL AXES OF ROTATION.

As the problem, *to determine the principal axes of rotation*, has not been solved in a satisfactory manner, by any mathematician or writer on mechanics; before clearing up the difficulty, we propose to exhibit it in a clear light without the least disguise.

A system being referred to three rectangular axes OX, OY, OZ, Fig. XVIII, if we call dM the element of the mass, and put $dM = dxdydz$; and further put

$$\int x^2dM = A; \quad \int y^2dM = B; \quad \int z^2dM = C;$$
$$\int xydM = D; \quad \int xzdM = E; \quad \int yzdM = F;$$

the moment of inertia of the system, in respect to the axis of OX, will be $\int (y^2 + z^2)\, dM = B + C$; in like manner, the moments of inertia, for the other two axes OY, OZ, will be $A + C$ and $A + B$, respectively. Let $OLPHDQAB$ be a rectangular parallelopiped, $OB = x$, $BD = y$, $DQ = z$; δ being put to represent the element of the mass, dM, or $dxdydz$. $BD^2 + DQ^2 =$ the square of the diagonal BQ the perpendicular distance of Q from the axis OX; hence $(y^2 + z^2)\delta$, will be the moment of inertia of the element δ, with with respect to the axis OX. The moments of inertia, for the three axes OX, OY, OZ, being given, it is required to find the moment of inertia for any other axis OK drawn through the plane $ABDQ$ and the origin O. Let ON be the projection of OK on the plane YOX, that is the plane $DHOX$; and let the angle NOX, on this plane be put $= \theta$, and the vertical angle $NOK = \phi$.

In the plane $DHOX$, draw DE perpendicular to ON, and let the co-ordinates be changed, by making $OE = x_1$, $DE = y_1$, $DQ = z_1, = z$. Complete the rectangle $QDEF$, the plane of which will also be perpendicular to the plane $DHOB$ and the line EF will be in the vertical plane NOK. Draw FC perpendicular to OK; complete the rectangle $QFCG$; we have $CG = FQ = ED = y_1 = y_2$. Again changing the co-ordinates, make $OC = x_2$; $CG = y_2$; $GQ = z_2$.

$CG^2 + GQ^2 =$ the square of the diagonal QC, the perpendicular distance of Q from the axis OK; hence $(y_2^2 + z_2^2)\delta$, will be the moment of inertia of the element δ, with respect to the axis OK.

On the plane $DHOB$, we have two right-angled triangles OBD, OED, with a common hypothenuse, the diagonal OD; for clearness this diagonal is not drawn in Fig. XVIII, but, this line is shewn in the plan, Fig. XIX. Put OD, Fig. XIX $= p$; then,

$$\frac{y}{p} = \sin DOB; \ \frac{x}{p} = \cos DOB; \ \frac{y_1}{p} = \sin (DOB - \theta);$$
$$\text{and } \frac{x_1}{p} = \cos (DOB - \theta.)$$

Now,

$$x_1 = p \cos DOE = p \cos (DOB - \theta) = p\left(\frac{x}{p} \cos \theta + \frac{y}{p} \sin \theta\right) = x \cos \theta + y \sin \theta;$$

$$y_1 = p \sin DOE = p \sin (DOB - \theta) = p\left(\frac{y}{p} \cos \theta - \frac{x}{p} \sin \theta\right) = y \cos \theta - x \sin \theta.$$

To render these expressions less cumbersome, put v for $\sin \theta$, and v_1 for cosine θ; and for the sake of uniformity call OE, ED, DQ, Fig. XXII, x_1, y_1, z_1, respectively. Then,

$$\left.\begin{aligned} OE &= x_1 = xv_1 + yv \\ ED &= y_1 = yv_1 - xv \\ DQ &= z_1 = z \end{aligned}\right\}; \quad (N); \left\{\begin{array}{l}\text{The second set of co-ordinates} \\ x_1, y_1, z_1, \text{ in terms of } \theta \text{ and the} \\ \text{first set of co-ordinates } x, y, z.\end{array}\right.$$

On the vertical plane $NKEOFO$, we have two right-angled triangles OEF, OCF, with a common hypothenuse OF, which, for the sake of clearness, is not drawn in Fig. XVIII, but this line is given in the elevation of $OECF$, Fig. XX.

Put OE, Fig XX, $= q$;

$$\text{then, } \frac{z}{q} = \sin EOF; \ \frac{x_1}{q} = \cos EOF.$$

$$\frac{z_2}{q} = \sin (FOE - \phi); \text{ and } \frac{x_2}{q} = \cos (FOE - \phi).$$

Then,

$$x_2 = q \cos FOC = q \cos (FOE - \phi) = q\left(\frac{x_1}{q} \cos \phi + \frac{z}{q} \sin \phi\right) = x_1 \cos \phi + z \sin \phi;$$

$$z_2 = q \sin FOC = q \sin (FOE - \phi) = q\left(\frac{z}{q} \cos \phi - \frac{x_1}{q} \sin \phi\right) = z \cos \phi - x_1 \sin \phi.$$

For the sake of conciseness and uniformity, put u for sin ϕ and u_1 for cos. ϕ ; and calling OC, OG, GQ, Fig. XVIII, x_2, y_2, z_2, respectively, we have

$$\left.\begin{array}{l} OC = x_2 = x_1u_1 + zu \\ CG = y_2 = y_1 \\ GQ = z_2 = zu_1 - x_1u \end{array}\right\} ; \quad (P) ; \left\{\begin{array}{l} \text{The third set of co-ordinates} \\ x_2,\ y_2,\ z_2, \text{ in terms of } \phi,\ \theta \\ \text{and } x_1,\ y_1,\ z_1. \end{array}\right.$$

Substituting in (P) the values of x_1, y_1 given in (N) we obtain,

$$\left.\begin{array}{l} OC = x_2 = (xv_1 + yv)u_1 + yv \\ CG = y_2 = yv_1 - xv \\ GQ = z_2 = zu_1 - (xv_1 - yv)u \end{array}\right\} ; \quad (Q) ; \left\{\begin{array}{l} \text{The third set of co-} \\ \text{ordinates } x_2,\ y_2,\ z_2, \\ \text{in terms of the} \\ \text{given co-ordinates} \\ x,\ y,\ z, \text{ and the} \\ \text{variables } \theta \text{ and } \phi. \end{array}\right.$$

Putting Σ for the moment of inertia with respect to the axis OCK, and deducing the value of $y_2^2 + z_2^2$ from (Q), we obtain,

$$\Sigma = \int (y_2^2 + z_2^2) = A(v^2 + v_1^2u^2)$$
$$+ B(v_1^2 + v^2u^2) + Cu_1^2 - 2Dvv_1u_1^2 - 2Ev_1uu_1^2 - 2Fvuu_1; \quad (R)$$

With the data given in equations, (Q) we can determine $\int x_2y_2dM$; $\int x_2z_2dM$; and comparing their values with that of Σ, (R), supposed to be already found, we shall have,

$$\int x_2y_2dM = -\frac{1}{2u_1}\left(\frac{d\Sigma}{d\theta}\right);$$

and
$$\int x_2z_2dM = -\frac{1}{2}\left(\frac{d\Sigma}{d\phi}\right);$$

It must not be forgotten that A, B, C, &c., in equation (R), represent the given quantities $\int x^2dM$, $\int y^2dM$, $\int z^2dM$, &c., which we have before specified.

At present we pay but little attention to devices for lessening the labour of the calculator; the principal objects in view, being to render the process of reasoning easily intelligible and clear, and to present it in as simple a form as possible that will not restrict generalisation.

Amongst all the axes drawn through the origin O, Fig. XVIII, let it be required to find an axis OK, to which, if the moments of inertia of the system be referred, it shall be a maximum or a minimum. This problem can be solved from the two equations.

$$\left(\frac{d\Sigma}{d\theta}\right) = 0; \text{ and } \left(\frac{d\Sigma}{d\phi}\right) = 0.$$

Differentiating, therefore, the value of Σ, equation (R), separately, with respect to the two variable quantities θ and ϕ, and putting both the differentials equal to nothing, we shall have two equations to determine the angles θ and ϕ, that fix the position of the angle required. When ϕ is eliminated, there will remain, to determine the angle θ, a complicated cubic equation of the form

$$A_1 \tan^3 \theta + B_1 \tan^2 \theta + C_1 \tan \theta + D_1 + 0; \qquad (S).$$

To solve the ultimate cubic equation has, up to the present time, presented such difficulties, that no one, in any practical case, attempted the solution. Some writers, by peculiar devices, have put this ultimate cubic equation into a simpler form than other writers, yet the difficulty remained. Cardan's rule will not apply, and the methods of Sturm and Horner are too laborious and uncertain, especially when the roots are nearly equal and represented by very large numbers. Equation (S) must have one real root, it being a cubic; and since the moment of inertia for every axis is a positive quantity (S), must have two roots. Again, if there be one axis which gives the moment of inertia a maximum, there must necessarily be another which gives it a minimum, and *vice versâ*. But if two roots be real, the third also must be real. Then (S) will have all its three roots real, and hence, generally speaking, will indicate three axes. M. Poisson gives the final cubic (S) under the form $\{gg' - hg' - (f'h' + hf' - ff' + g'h')\, u\} \times \{h'(1 - u^2) + (f - g)\, u\} + (g'u + f')(f'u - g')^2 = O$; ($T$). In which

$$\text{tangent } \theta = \frac{(f'u - g'\sqrt{1 + u2})}{h'(1 - u^2) + (f - g)\, u}.$$

In this equation (T), u is the unknown quantity and has three positive values, each of which gives the sine of two angles, one less than $\frac{\pi}{2}$, and the other its supplement. By dual arithmetic, in any cubic equation, the value of the unknown quantity may be found in a few minutes under an endless variety of forms, but all reducible to one or other of the roots. The labour and difficulty experienced in finding the roots of equations higher than a quadratic, were such that writers on mechanics and other branches of applied mathematics were prevented from introducing problems involving cubic or higher equations. Nevertheless such eqations as (S), and (T), may be readily solved, we will now find the roots of a cubic equation, the most difficult that can be imagined; the co-efficients of this equation are expressed by very large numbers, and its roots are all positive and nearly equal.

Given, $20412203{\cdot}40404u^3 - 43842278{\cdot}4033752u^2 + 31357374{\cdot}5709922u - 7478846{\cdot}03569 = O$; (W); to find the three roots.

The solution of the equation (W), may be effected by an ordinary calculator in a few hours. This method will be explained hereafter. In the present example we find,

$$u = +\ {\cdot}727374509 81487;\ u = +\ {\cdot}71299103200788;$$
$$\text{and } u = +\ {\cdot}70748436524477.$$

The accuracy of each of these roots may be tested to seven places of figures by the use of an ordinary table of logarithms.

Unless the axis, around which the vessel turns, be accurately determined, not assumed, as has been the case, nothing but error and danger can result.

TO FIND THE MOMENT OF INERTIA OF AN ELLIPSOID WITH RESPECT TO THE THREE PRINCIPAL AXES.

Let O be the origin of three rectangular co-ordinates OX, OY, OZ; $CDAB$ an ellipse in the plane of XOZ, whose centre is at O; $CEAF$ an ellipse in the plane of XOY; and $BEDF$ an ellipse in the plane of YOZ, whose centres are also at O. We may conceive a solid termed an *ellipsoid* to be formed by an ellipse $AECF$ moving parallel to itself, its two axes at the same time so varying, that it shall slide along another ellipse $EDFB$, like the ellipse *mrns*, Fig. XXI, whose centre has moved from O to p, along the line OD.

Putting $OC = a = OA$; $OE = b = OF$; $OB = c = OD$.

Let δ be a point in the surface of this *ellipsoid*, and $Og = x$, $gh = y$, and $h\delta = z$; and completing the rectangular parallelopiped, we have, $BO^2 : OC^2 :: Bp \times pD : pn^2$, from a well known property of the ellipse; that is,

$$c^2 : a^2 :: (c^2 - z^2) : \frac{a^2\,(c^2 - z^2)}{c^2} = (pn)^2.$$

Again, in the ellipse $BEDF$, we have, $BO^2 : OE^2 :: Bp \times pD : (pr)^2$,

$$\text{or, } c^2 : b^2 :: (c^2 - z^2) : \frac{b^2\,(c^2 - z^2)}{c^2} = (pr)^2.$$

Now, in the ellipse $rmsn$, we have, $(pn)^2 = \dfrac{a^2\,(c^2 - z^2)}{c^2} = (pm)^2$; $(pr)^2 = \dfrac{b^2\,(c^2 - z^2)}{c^2} = (ps)^2$; and $pf = x$; $f\delta = y$; therefore,

$$\frac{a^2\,(c - z^2)}{c^2} : \frac{b^2\,(c^2 - z^2)}{c^2} :: \frac{a^2\,(c^2 - z^2)}{c^2} - x^2 : y^2; \text{ that is,}$$

$$a^2 : b^2 :: \frac{a^2\,(c^2 - z^2)}{c^2} - x^2 : y^2.$$

$$\therefore \frac{a^2b^2\,(c^2 - z^2)}{c^2} - b^2x^2 = a^2y^2,$$

and $\therefore a^2b^2c^2 = a^2b^2z^2 + a^2c^2y^2 + b^2c^2x^2$, and hence,

$$\frac{z^2}{c^2} + \frac{y^2}{b^2} + \frac{x^2}{a^2} = 1; \qquad (S).$$

This is the equation of the surface of the ellipsoid $ABCnDmrEBFD$, and holds good for every point δ on the surface.

By integrating $(x^2 + y^2)\,dzdydx$ with respect to z, there results $(x^2 + y^2)\,zdydx$ + constant.

From equation (S), $z = \pm\, c\sqrt{1 - \frac{y^2}{b^2} - \frac{x^2}{a^2}}$, which furnishes the two limits of z. The definite integral will consequently be

$$2\rho c x^2\sqrt{1 - \frac{y^2}{b^2} - \frac{x^2}{a^2}}\,dxdy + 2\rho c y^2\sqrt{1 - \frac{y^2}{b^2} - \frac{x^2}{a^2}}\,dxdy, \quad (T);$$

ρ being put for the density of the *ellipsoid*, which is supposed to be constant. In order to abridge, make $b^2 - \dfrac{b^2x^2}{a^2} = r^2$; then the integral, relative to y, of the first part of (T), will become

$$\frac{2\rho c x^2 dx}{b}\int\sqrt{r^2 - y^2}\,dy; \qquad (Q).$$

From the equation of the section $AECF$ of the ellipsoid, on the plane XOY, we have, $\dfrac{y^2}{b^2} + \dfrac{x^2}{a^2} = 1$,

$$\text{therefore, } y = \pm\sqrt{b^2\left(1 - \frac{a^2}{x^2}\right)} = \pm\sqrt{b^2 - \frac{b^2x^2}{a^2}}.$$

It appears also, that $r = \pm \sqrt{b^2 - \frac{b^2x^2}{a^2}}$; consequently, it is clear that $r = \pm y$ are the two limits of the integral (Q), relative to y.

It will now be shown that $\int_r^r \sqrt{r^2 - y^2}\, dy = \frac{\pi}{2} r^2$; ($P$).

Since $\sqrt{r^2 - y^2} = \frac{r^2 - y^2}{\sqrt{r^2 - y^2}}$, we obtain by making $y = rv$,

$$dy = rdv \text{ and } y^2 = r^2v^2.$$

By taking the first member of (P) and substituting these results,

$$\int_{-r}^{r} \sqrt{r^2 - y^2}\, dy = \int_{-r}^{r} \frac{r^2 - y^2}{\sqrt{r^2 - y^2}}\, dy = \int_{-r}^{r} \frac{r^2 - r^2v^2}{\sqrt{r^2 - r^2v^2}}(rdv)$$

$$= \int_{-r}^{r} \frac{r^2dv}{\sqrt{1 - v^2}} - \int_{-r}^{r} \frac{r^2v^2dv}{\sqrt{1 - v^2}}; \quad (R).$$

But since during the process of integration r is supposed to be constant,

$r^2\int_{-r}^{r} \frac{dv}{\sqrt{1 - v^2}} = r^2 \times$ (an arc whose sine$=v$) taken between the limits $+ r$ and $- r$; but $v = \frac{y}{r}$,

$\therefore r^2 \int \frac{dv}{\sqrt{1 - v^2}} = r^2 \sin^{-1}\frac{y}{r}$; which becomes $r^2 \sin^{-1} 1$, when $y = r$.

But an arc whose sine is 1, $= \frac{\pi}{2} \therefore r^2 \int_{-r}^{r} \frac{dv}{\sqrt{1 - v^2}} = r^2\pi$.

From the second member of (R),

$\frac{r^2v^2dv}{\sqrt{1 - v^2}} = r^2 \frac{v^2\, dv}{\sqrt{1 - v^2}}$, the integral of which $= r^2 \left(- \frac{1}{2} v \sqrt{1 - v^2} + \frac{1}{2} \sin^{-1} v.\right)$ when taken between the limits $+ r$ and $- r$, after $\frac{y}{r}$ is substituted for v, becomes $\frac{1}{2} \pi r^2$.

Consequently $\frac{2\rho cx^2dx}{b}\int\sqrt{r^2 - y^2}\, dy = \frac{\rho\pi cr^2x^2dx}{b}$, which, by substituting for r^2, its value, $b^2 - \frac{b^2}{a^2} x^2$ and reducing, becomes $\frac{\rho\pi bc}{a^2} (a^2x^2 - x^4)\, dx$, the integral of which is $\frac{\rho\pi bc}{a^2}\left(\frac{a^2x^3}{3} - \frac{x^5}{5}\right)$

and when taken between the limits $x = +a$, $x = -a$, there results

$$\frac{2\rho\pi bc}{a^2}\left(\frac{a^5}{3} - \frac{a^5}{5}\right) = \frac{4\rho\pi bca^3}{15}.$$

$$\therefore 2\rho c \iint x^2 \sqrt{1 - \frac{y^2}{b^2} - \frac{x^2}{a^2}}\, dydx = \frac{4\pi\rho a^3 bc}{15}.$$

A similar process of reasoning will apply to

$$2\rho c y^2 \sqrt{1 - \frac{y^2}{b^2} - \frac{x^2}{a^2}}\, dxdy,$$

the other member of (T), the expressions being symmetrical, we shall have, without further investigation,

$$2\rho c \iint y^2 \sqrt{1 - \frac{y^2}{b^2} - \frac{x^2}{a^2}}\, dxdy = \frac{4\pi\rho b^3 ac}{15},$$

Therefore the values of the moment of inertia with respect to the axis of z, will be $\frac{4\pi abc}{15}(a^2 + b^2)$ the total integral of (T). In a similar manner, the moments of inertia with respect to the axes of x and y, may be found. If M, the mass of ellipsoid, $= \frac{4\pi\rho abc}{3}$, then, the moment of inertia with respect to the diameter $DB = 2c$, will be represented by

$$\frac{1}{5} M (a^2 + b^2).$$

The moment of inertia for $EF = 2b$, $\frac{1}{5} M (c^2 + a^2)$, and for $AC = 2a$, the moment of inertia $= \frac{1}{5} M (c^2 + b^2)$. Now if a be greater than b, and b be greater than c; the moment of inertia about AC will be less than the moment of inertia about either EF or BD. And the moment of inertia about DB will be greater than that about EF.

In order to form a design we have selected for the midship section a semi-ellipsis, not simply because it admits of more rapid and accurate calculation, but as, except a circle, it gives the largest area with the smallest periphery and least friction surface, while it is also the strongest form with equal material.

We also generally object to a greater rise of floor than this form entails, and we strongly object to flat floors, for the following reasons :—

1. They increase the area of midship section, therefore, the area of resistance without any adequate increase of the capacity.

2. Are more liable to damage from rocks.

3. Reduce the action of the rudder and screw.

4. In rolling their angles are protruded below the keel, and such ships besides requiring a greater draught of water are more liable to injury when entering a harbour when the water is not of much greater depth than they draw, as one of the angles is likely to take the ground at each roll.

5. Tend to reduce the stability, and on a wind they increase the wind-couple and reduce the practical stability still further.

6. They tend to take strains more proper for the keel; in a word, there is nothing to justify that form of midship body; any necessary capacity can be better obtained otherwise.

Greater ease of motion dictates that the depth should be equal to half the breadth, when it is less in that proportion there is a rise of the centre of gravity as the ship rolls; this tends to produce independent oscillations that are better avoided when it is feasible; this proportion cannot be preserved in cargo vessels, or steam vessels, arising from the great changes that take place in the amount of their cargo or coals; the less departure, however, the better for sea-going vessels. But when the depth of water is limited there is no choice, any necessary additional depth can be given.

For the form of the horizontal lines we have taken Scott Russell's wave lines, as we think they afford the greatest number of advantages, besides that being formed on a law, they admit of an equation which facilitates rapid and accurate calculation.

The proportion of length to breadth will depend upon what is required of the ship. If speed alone, great proportional length. If cargo carriage, a less proportion of length, and if necessary for this purpose, a straight of equal breadth might be introduced between the extremities. We have, however, selected fine lines to illustrate the system.

We add this further explanation—

We have given only four divisions and four water-lines, Figs. XXII, XXIII, XXIV; but when greater accuracy of form, &c., is required, the system pursued in Fig. XXV is followed.

Waves vary in their length.—Waves are found to vary in their length according to the velocity with which they move. The lengths corresponding to the velocities have been observed and tabulated, so that when the velocity at which a vessel shall be driven is determined, the length of the bow will be the length of wave in the table which corresponds with this velocity.

Propagation of the wave of oscillation.—If a plane be moved fast in water there will be left behind it a partial vacuum, which will principally be filled up by the water from below, because that will be forced in by the superincumbent water, while that at the surface will be so only by the force of the circumambient water, which force is very much less; the water then, which is immediately above that which is forced into the vacant space, will fall, and the consequence will be that an undulatory or oscillating motion will be produced; this motion has been called a wave of oscillation, and is that which is formed behind a vessel when she moves ahead; the length of this wave, as of that of the wave of translation, depends on the velocity of its propagation, which will depend upon the velocity of the moving body. The water being divergent from the bow, if the acceleration of it were continued up to the end of the bow there would be a partial vacuum formed, then the vessel would sink, and the resistance would be increased, so there would be a loss of power; such, however, is not the case aft, as the water from the after-body is convergent, and therefore may be accelerated up to the sternpost with advantage, for the greater its velocity there the greater will be its reaction, which will be favourable to the progress of the vessel as giving her an onward thrust. So it may seen from Fig. XXV, that the after-body is only half the length of the wave, the body terminating when the wave is at its highest point and before it has subsided.

Comparative length of the waves of oscillation.—The length of the wave of translation as compared with the length of the wave of oscillation is as 2 to 3, but as only half of the wave of the latter is taken, but the whole of the former, so the length of the fore-body as compared with the length of the after-body is as 3 to 2.

The genesis of the wave-line curve forward.—Fig. XXV is a theoretic wave curve of a water-line; the genesis of these curves is as follows—the length of the fore-body as compared with the length of the after-body is as 3 to 2, therefore the whole length is divided into 5 equal parts, and 3 allotted to the fore-body. A circle, whose diameter is equal to the half breadth determined upon, is described with its circumference touching the central line, where the fore and after-bodies join; its circumference is divided into sixteen equal parts, and the central lines of the fore and after-bodies are each divided into eight equal parts; then, for the curve of the fore-body, from the foremost division on the central line lay off the perpendicular distance of the central line from the first or lowest division on the circumference of the circle, and from the second division on the central line the perpendicular distance of the second division on the circle, and so of each of the eight divisions; then through these points draw a line, and it will be the wave-line curve forward. The curves of all the water-lines are similar.

The genesis of the wave curve for the after-body.—For the after-body, lines are drawn from the divisions on the circle parallel to the central line, on which the distances of the divisions on the central line from the fore end of the after-body are respectively laid off from the divisions on the circle, a line drawn through these points will give the wave curve for the after-body. The curves of the other water-lines are similar.

Let Fig. XXVI represent the immersion portion of the body of a wave-line vessel.

Fig. XXVII represents the curves of a water-line, constructed in accordance with the wave-line low, each of the lower water-lines are constructed in the same manner, the equation for the curve may be determined as follows—

The point C describes the curve $CC_1C_2C_3 \ldots V$ as the semicircle CDO, Fig. XXVII slides and turns uniformly in passing along the line OV. When C has moved until it is over $\frac{2}{6}$, or $\frac{1}{3}$ the line OV; Q_2 shows the position of the semicircle; when C has moved over $\frac{4}{6}$, or $\frac{2}{3}$ of the line OV, Q_4 shows the position of the generating semicircle; $OF_2 = \frac{2}{6}\,OV$; $F_2C_2 = b$, the radius of the circle CDO, plus the cosine of $\frac{2}{6}$ of the semicircle ODC, to radius b. And $OF_4 = \frac{2}{6}\,OV$; $F_4C_4 = b +$ the cosine of $\frac{4}{6}$ of the semicircle, to radius b; it may be necessary to observe, that when the fractional part of OV be greater than $\frac{1}{2}$, the cosine becomes negative.

If F_n be put to represent any point on the line OV, and C_n the corresponding point on the curve $CC_1C_2C_3$ . . . . then, by putting $OV = a$, $OF_n = x$, and $F_nC_n = y$; it is easily perceived, that $y = b + b \cos \frac{\pi x}{a}$, π as is usual, being put for the length of an arc of a semicircle whose radius $= 1$.

Hence it follows that $x = \frac{a}{\pi} \cos^{-1}\left(\frac{y-b}{b}\right)$; when $\frac{b-y}{y}$ is negative, the corresponding arc to this cosine, written $\cos.^{-1}\left(\frac{y-b}{b}\right)$, is greater than $\frac{\pi}{2}$.

Fig. XXVIII represents a quarter of the fore-body, the principal section of which is half a semi-ellipsis.

Fig. XXIX, a quarter of the after-body, which also has half a semi-ellipsis for its principal section.

PROBLEM I.

To find the area of the fore-deck plane CVB, Fig. XXVII, which passes horizontally through the greatest breadth BC, $= 4b$.

Since $y = b + b \cos \frac{\pi x}{a}$; and as the circle CDO moves and turns in passing from O to V, the co-ordinate x of the curve CC_1C_2 . . . . increases, but its co-ordinate y diminishes; hence the differential of y will be negative, and therefore

$$- dy = b \sin \frac{\pi x}{a} \, d\left(\frac{\pi x}{a}\right).$$

Then as the general expression for the area $= \int x dy$, we have

$$\text{Area of } CC^1C_2 \,..\, VO = b \int \sin\left(\frac{\pi x}{a}\right) d\left(\frac{\pi x}{a}\right) x =$$

$$\frac{ab}{\pi} \int \left(\frac{\pi x}{a}\right) \sin\left(\frac{\pi x}{a}\right) d\left(\frac{\pi x}{a}\right).$$

But, generally, X being any function of x, we have

$\int X \sin X dX = \sin X - X \cos X$; consequently,

$$\text{Area} = \frac{ab}{\pi}\int \left(\frac{\pi x}{a}\right) \sin\left(\frac{\pi x}{a}\right) d\left(\frac{\pi x}{a}\right) =$$

$$\frac{ab}{\pi}\left\{ \sin\left(\frac{\pi x}{a}\right) - \left(\frac{\pi x}{a}\right) \cos\left(\frac{\pi x}{a}\right)\right\} + \text{correction.}$$

We know that the area vanishes when $x = o$, hence no correction is required. Taking

$$\frac{ab}{\pi}\left\{ \sin\left(\frac{\pi x}{a}\right) - \left(\frac{\pi x}{a}\right) \cos \frac{\pi x}{a}\right\}$$

between the limits $x = o$ and $x = a$, we find $a \times b$ or ab, to be the area of OCC_1C_2 . . . VO, half the area of the fore-deck plane. Therefore the area of the whole plane $CVB = 2ab =$ the area of an isosceles triangle whose base is $CB = 4\,b$, and perpendicular height $OV = a$.

PROBLEM II.

To find the Area of the Aft Deck Plane $C, {}_1C, {}_2C, {}_3C$ . . . Z, Fig. XXVII which passes horizontally through the greatest breath $BC = 4b$.

The point C describes the curve $C_1C_2C_3C$ . . . Z as the semicircle CFO slides and turns uniformly in passing along the line OZ. ${}_2Q$ shews the position of the generating semicircle CEO for $\frac{2}{6}$, or $\frac{1}{3}$ of the line OZ; ${}_2CO_2 = b + b \cos \frac{\pi \times 2}{6}$, as in the plane of the fore deck, but $O_2O = \frac{2}{6}$ of OZ + the sine of $\frac{2}{6}$ the semicircle OEC to radius b. ${}_4Q$ shows the position of the generating semicircle CEO for $\frac{4}{6}$ of the line OZ; ${}_4CO_4 = b + b \cos \frac{\pi \times 4}{6} = b\left(1 + \cos \frac{\pi \times 4}{6}\right)$ to radius 1; observing, that when the fractional part of OZ be greater than $\frac{1}{2}$ the cosine becomes negative, but, and observe it well, $OO_4 = \frac{4}{6}$ of OZ + the sine of $\frac{4}{6}$ of the semicircle CEO to radius b.

If O_n be put to represent any point on the line OZ, and $_nC$ the corresponding point on the curve $C_1C_2C \; . \; . \; . \; . \; Z$, then by putting $OZ = c$, $OO_n = x$ and $O_n \, _nC = y$, it is apparent that $x = u + b \sin \frac{\pi u}{c}$; u being the portion of OZ passed over, and to which the sine of $\frac{u}{c}$ part of the generating semicircle is added.

$y = b\left(1 + \cos\frac{\pi u}{c}\right)$, as on the plane of the fore body.

$$\text{From } y = b + b\cos\frac{\pi u}{c}, \text{ we have } \frac{y-b}{b} = \cos\frac{\pi u}{c}, \text{ and}$$

$$u = \frac{c}{\pi}\cos^{-1}\left(\frac{y-b}{b}\right);$$

$$\text{and from } x = u + b\sin\frac{\pi u}{c},\ \frac{u-x}{b} = \sin\frac{\pi u}{c}.$$

$$\therefore \left(\frac{y-b}{b}\right)^2 + \left(\frac{u-x}{b}\right)^2 = 1,\ \therefore x - \sqrt{2by - y^2} = u.$$

$$\text{Hence } x = \sqrt{2by - y^2} + \frac{c}{\pi}\cos^{-1}\left(\frac{y-b}{b}\right).$$

Thence, as the general expression for the area $= \int x dy$;
Area of $C_1C_2C \ldots ZO =$

$$\int (2by - y^2)^{\frac{1}{2}} dy + \frac{c}{\pi}\int \cos^{-1}\left(\frac{y-b}{b}\right) dy = \int x dy.$$

$$\left.\begin{array}{l} \text{But } \frac{c}{\pi}\int \cos^{-1}\left(\frac{y-b}{b}\right) dy = \frac{cy}{\pi}\cos^{-1}\frac{y-b}{b} - \frac{c}{\pi}(2by - y^2)^{\frac{1}{2}} \\ \qquad + \frac{cb}{\pi}\text{ ver}^{-1}\frac{y}{b} \Big\} \\ \text{and } \int (2by - y^2)\, dy = \frac{y}{2}(2by - y^2)^{\frac{1}{2}} - \frac{b}{2}(2by - y^2)^{\frac{1}{2}} + \\ \qquad \frac{b^2}{2}\text{ ver}^{-1}\frac{y}{b}; \Big\} \end{array}\right\} \text{add}$$

$$\therefore \text{ Area} = \frac{cy}{\pi}\cos^{-1}\left(\frac{y-b}{b}\right) + \left(\frac{y}{2} - \frac{b}{2} - \frac{c}{\pi}\right)(2by - y2)^{\frac{1}{2}} + \left(\frac{b^2}{2} + \frac{cb}{\pi}\right)\text{ ver}^{-1}\frac{y}{b} + \text{correction}.$$

No correction is required, for the expression vanishes when $y = o$, but when taken between the limits $y = o$ and $y = 2b$, it becomes

$$\left(\frac{b^2}{2} + \frac{cb}{\pi}\right) \overset{-1}{\text{ver}}\, 2 = \frac{\pi b^2}{2} + cb,$$

the area of half the aft-deck plane, $= C_1C_2C_3C \ . \ . \ . \ . ZO$; whence, the area of the plane $C_2CZ_2BB = \pi b^2 + 2cb =$ the area of the generating circle $OECD$, and an isosceles triangle whose base $OB = 4b$, and perpendicular height $OZ = c$.

PROBLEM III.

To find the solid content of 1–4*th the fore solid, or as it is usually termed, the half fore body* $OHRQC_4XV$; and also to find the solid of any slice, $OQRSTX$, Fig. XXVIII, parallel to the plane QC_4XO.

$OHRQ$ is one quarter of the elliptic midship section;

$OH = h$; $OS = z$; $SR = 2z_1$; $OX = a$; $OQ = 2b$.

From a property of the ellipsis, $z_1 = \frac{b}{h}(h^2 - z^2)^{\frac{1}{2}}$; the area, as before shewn, Prob. I, of the plane $RST = az_1$; therefore, the solid content $= \int (az_1)dz$.

$$\text{But} \int (az_1)\, dz = \frac{ab}{h}\int (h^2 - z^2)^{\frac{1}{2}} dz = \frac{abz}{2h}(h^2 - z^2)^{\frac{1}{2}} + \frac{abh}{2} \sin \theta^{-1} \frac{z}{h} + \text{correction.} \qquad (J).$$

We know that the solidity should vanish when $z = 0$; now this happens in the above expression for the solid content of QHX when $z = 0$, hence, no correction is required.

When $z = h$, then $\frac{abz}{2h}(h^2 - z^2)^{\frac{1}{2}} + \frac{abh}{2} \overset{-1}{\sin} \frac{z}{h}$ becomes $abh \times \frac{\pi}{4}$.

Consequently, four such solids, as that shown in Fig. XXVI, would have a solid content $= 4bh \times \frac{\pi}{4} \times a$, which is equal to half the solid content of a cylindrical body whose base is an ellipse (conjugate diameter $= 4b$, transverse $= 2h$), and length $= a$.

Let p = a number greater than 1; and find an arc θ to radius 1, whose sine $= \frac{1}{p}$ and call q its cosine. Putting $z = \frac{h}{p}$ in (J), we have, for the solid content of any slice $QRST$,

$$\frac{abh}{2}\left\{\frac{\left(1-\frac{1}{p^2}\right)^{\frac{1}{2}}}{p} + \sin\ \frac{1}{p}\right\} = \frac{abh}{2}\left\{\frac{q}{p}+\theta\right\}; \qquad (K).$$

If $\frac{1}{p} = \frac{1}{3} = \cdot 3333333$; then $\theta = \cdot 3398371$; and q or $\cos\theta$ $= \cdot 9428091$.

$\therefore abh\left(\frac{\cdot 9428091}{3} + \cdot 3398371\right) = abh \times (\cdot 6541068)$ the solid content of the slice across the whole deck in front of the line OQ.

But $abh \times \frac{\pi}{2}$ = the solid content of the fore body from the greatest breadth to the keel, hence $abh\left\{\frac{\pi}{2} - \cdot 6541068\right\} =$ $= abh\ \{1\cdot 5757963 - \cdot 6541068\} = abh\ \{\cdot 9216895\}$ = the solid content of the lower part $RHSTV$, after the slice is removed. And further, $abh\ \{1\cdot 5757963 + \cdot 6541068\} = abh\ \{2\cdot 2299031\}$ = the solid content of twice $QRHOXV$, with twice the solid content of the slice $QRSOXT$ added. Suppose the vessel to be carried up square from the plane OQC_4X to a height $= \frac{h}{p}$, then the solid content of the part added would be $= 2ab \times \frac{h}{p} = \frac{2abh}{p}$; and the solid content of the fore body so formed would be $abh \times \frac{\pi}{2}$ + $abh \times \frac{2}{p} = abh\left\{\frac{2}{p}+\frac{\pi}{2}\right\}$; $\qquad (L)$.

PROBLEM IV.

To find the solid content of one-fourth the aft solid, Fig. XXIX, termed the aft half body, $OQRHFC_4C$; and also to find the solid content of any slice $QRSODC$, parallel to the plane Q_4COO.

The plane $_4CQOC$, Fig. XXIX, represents the plane $C_1C_2C_3C$.. ZO, Fig. XXVII, $OHRQ$ represents one quarter of the elliptic midship section; $OH = h$; $OS = z$; $SR = 2z_1$, $OC = c$; $OQ = 2b$; then $z_1 = \frac{b}{h}(h^2 - z^2)^{\frac{1}{2}}$ from a property of the ellipsis. The area of the plane passing through $DSR = cz_1 + \frac{\pi z_1^2}{2}$; see Problem II.

Therefore, the solid content of $QRHOC_4F$ will be expressed by $\int\left(cz_1 + \frac{\pi z_1^2}{2}\right)dz$, which becomes, $\int\left\{\frac{bc}{h}(h^2 - z^2)^{\frac{1}{2}} + \frac{\pi b}{2h^2}(h^2 - z^2)\right\}dz$ when the value of z_1 is substituted.

$$\frac{\pi b^2}{2h^2}\int(h^2 - z^2)\,dz = \frac{\pi b^2}{2h^2}\left(h^2 z - \frac{z^3}{3}\right) = \frac{\pi b^2 z}{2} - \frac{\pi b^2 z^3}{6h^2}; \text{ and,}$$

$$\frac{bc}{h}\int(h^2 - z^2)^{\frac{1}{2}}\,dz = \frac{bc}{h}\left\{\frac{z}{2}(h^2 - z^2)^{\frac{1}{2}} + \frac{h^2}{2}\sin^{-1}\frac{z}{h}\right\},$$

$$\text{consequently} \int\left\{\frac{bc}{h}(h^2 - z^2)^{\frac{1}{2}} + \frac{\pi b^2}{2h^2}(h^2 - z^2)\right\}dz =$$

$$\frac{bc}{h}\left\{\frac{z}{2}(h^2 - z^2)^{\frac{1}{2}} + \frac{h^2}{2}\sin\theta\frac{z}{h}\right\} + \frac{\pi b^2 z}{2} - \frac{\pi b^2 z^3}{6h^2}.$$

Hence the solid content of the solid $Q_4CCFHO =$

$$\frac{bc}{h}\left\{\frac{z}{2}(h^2 - z^2)^{\frac{1}{2}} + \frac{h^2}{2}\sin^{-1}\frac{z}{h}\right\} + \frac{\pi b^2 z}{2} - \frac{\pi b^2 z^3}{6h^2} + \text{correction}; (B).$$

When $z = 0$, the solid content $= 0$, and, therefore, the expression (B) for the solidity, if complete, must vanish when $z = 0$. It is found by inspecting (B) that the expression vanishes when $z = 0$; consequently (B) requires no correction. When $z = h$, (B) becomes.

$$\frac{bc}{h} \times \frac{h^2}{2} \times \frac{\pi}{2} + \frac{\pi b^2 h}{2} - \frac{\pi b^2 h^3}{6h^2} = \frac{\pi bch}{4} + \frac{\pi b^2 h}{3} =$$

$$\pi bh\left(\frac{b}{3} + \frac{c}{4}\right); \quad (\gamma).$$

Twice the solid QHC, or the aft body from the keel up to the greatest deck plane, has a solid content represented by $2\pi bh\left(\frac{b}{3} + \frac{c}{4}\right)$, which is equal to a cylindrical solid whose base is the whole ellipse, of which $OQRH$ is the quarter, and whose length is equal $\frac{1}{12}$ of (twice OQ + three times OC).

The content of the whole solid, that is four times the solid QHC, $= \pi bch + \frac{4}{3} b^2h$ = half a cylindrical solid whose base is the ellipse of which $OQRH$ is the quarter, and whose length is equal $OC = c$; together with a spheroid, the revolving axis being $2b = OQ$, and the fixed axis $2h$ = twice OH.

Now if

$$z = OS = \frac{h}{p}. \ (B) \text{ becomes } \frac{bch}{5}\left\{\frac{1}{p}\left(1-\frac{1}{p^2}\right)^{\frac{1}{2}} + \sin^{-1}\frac{1}{p}\right\} + \frac{\pi b^2h}{2}\left\{\frac{1}{p} - \frac{1}{3p^3}\right\} \quad (J);$$

the solid content of the slice $ORSOCD$. Suppose $p = 4$, then $\frac{1}{p} = {\cdot}2500000$. If θ be put for an arc, to radius 1, whose sine $= \frac{1}{p}$; $\left(1-\frac{1}{p^2}\right)^{\frac{1}{2}} = \cos\theta$, which call q; then, (J), may be put under the form

$$\frac{bch}{2}\left\{\frac{q}{p} + \theta\right\} + \frac{\pi b^2h}{2}\left[\frac{1-\frac{1}{p^2}}{p}\right]; \qquad (K).$$

In the example we have chosen, $\theta = {\cdot}2526801$, an arc of $14^\circ \,..\, 28' \,..\, 39''$ to radius 1; $\cos\theta = {\cdot}9682458 = q$; then, in this case (K) becomes,

$$\frac{bch}{2}\left\{\frac{{\cdot}9682458}{4} + {\cdot}2526801\right\} + \frac{\pi b^2h}{2}\left[\frac{1-\frac{1}{16}}{4}\right] =$$

$$\frac{bch}{2}\left\{{\cdot}4947416\right\} + \frac{\pi b^2h}{2}\left[\frac{15}{64}\right].$$

Consequently, the solid content of the slice, across the aft half body cut by a plane DSR parallel to the plane of COQ, whose depth

$$OS = \frac{1}{4} \text{ of } OH = bch\{{\cdot}4947416\} + \pi b^2h\left[\frac{15}{64}\right].$$

If the slice be removed the solid content of the remaining portion below the plane DSR will be

$$bch\left\{\frac{\pi}{2} - {\cdot}4947416\right\} + b^2h\left[\frac{2\pi}{3} - \frac{15}{64}\right] =$$

$$bch\,(1{\cdot}0760547) + b^2h\,[1{\cdot}8600201] \qquad (L).$$

But if the slice be added above the plane QOC the solid content of the body thus formed will be

$$bch\left\{\frac{\pi}{2} + {\cdot}4947416\right\} + b^2h\left[\frac{2\pi}{3} + \frac{15}{64}\right].$$

METHOD OF AVERAGING PRODUCTS, AND OF FINDING THE CENTRES OF GRAVITY AND GYRATION OF COMPOUND BODIES LIKE SHIPS, WITHOUT CALCULATION.

Given two rectangular parallelograms $aefg$ and $ebhn$, to find a rectangular parallelogram $adcb$ whose area is equal to the sum of the areas $agfe$, Fig. XXXII, and $ebhn$ together.

Produce gf to B, draw the diagonal AB cutting fn in q; draw dqc parallel to Ah, then the rectangular figure $dcba = aefg + ebhn$.

For the rectangle $qnhc$, according to the proposition before proved, page 69, Fig. II, is equal to the rectangle $qdgf$.

This proposition establishes the truth of the method here introduced to average, almost instantly, any number of products without calculation. The following simple example will illustrate the method.

SEE TABLE XXXIII.

A weight of	80 lbs.	distance	of its	centre of	gravity from	a fixed plane	6 feet.	
,,	25 ,,	,,	,,	,,	,,		11	,,
,,	65 ,,	,,	,,	,,	,,		15	,,
,,	40 ,,	,,	,,	,,	,,		17	,,
,,	50 ,,	,,	,,	,,	,,		18	,,
,,	50 ,,	,,	,,	,,	,,		21	,,
,,	70 ,,	,,	,,	,,	,,		23	,,
,,	80 ,,	,,	,,	,,	,,		26	,,
,,	60 ,,	,,	,,	,,	,,		31	,,
,,	60 ,,	,,	,,	,,	,,		37	,,
,,	90 ,,	,,	,,	,,	,,		40	,,
,,	50 ,,	,,	,,	,,	,,		43	,,
,,	50 ,,	,,	,,	,,	,,		47	,,
,,	60 ,,	,,	,,	,,	,,		52	,,

Required the distance of the centre of the centre of gravity of all these weights from the same fixed plane without calculation.

Answer 28⅛ feet.

The mode of using this table is as follows:—Fig. XXXIII.

A horizontal line is drawn through the point *W* till it cuts the perpendicular to the second weight at *a*, then a line is drawn from 11 feet, the distance of second weight. A horizontal line, dotted and drawn to and through *b*, the point of intersection of the perpendicular line, the first weight and the above diagonal gives the number of feet, which is the average required for the two first weights. The process may be continued, and *b*3, or *b*7, or *b*10, or *b* 13 is the average for all the weights preceding any one that may be selected.

The lines on the calculating scale are not necessary; we may suppose a large slate, like *ABCD*, having very small accurate squares cut over its surface, numbered at the top and left side. That constituting a measure that may be employed for obtaining various averages.

APPENDIX B.

MR. FROUDE'S ERROR AS TO THE ROLLING OF SHIPS.

As Mr. Froude's theories concerning the best mode of preventing the rolling of ships in a seaway have been extensively applied in our Navy, it has become a matter of great importance to demonstrate their erroneous character, and to point out the great danger of continuing to apply them.

It is fortunate that the persons are few who have, as yet, adopted them, and these few, for the most part, amateurs, unacquainted with the behaviour of ships at sea; however, as these have been, and are being, allowed to experiment, without risk to themselves, but to the great peril of the lives of our officers and seamen, the case has become very serious.

Let us now examine the principles on which Mr. Froude proceeds.

He alleges that "the effort of stability is the lever by which a wave forces a ship into motion; if a ship were destitute of this stability," he says, "no wave that the ocean produces would serve to put her in motion."

He offers the following explanation and illustration in proof of the correctness of the above view:—

"I regret you did not notice the conclusive experimental proof I exhibited of the fundamental proposition, that the surface of a fluid when dynamically inclined is virtually level to a body which floats on it, and as it really lies at the root of the whole theory of the behaviour of a ship amongst waves, I am anxious that not only my re-statement of it, but reference to the experimental proof of it should appear also."

Mr. Froude then gives the following as the experimental proof of his fundamental proposition. If this, therefore, is shown to be without foundation in truth, his whole theory falls to the ground.

"If a shallow dish of water be suspended by three equal strings brought to one point, so that it is level when it hangs at rest, then if the meeting of the strings be taken in the fingers, the dish may be swung about in any direction whatever without spilling the water; indeed, it is impossible to spill the water so long as the suspending lines are kept tight by the operation; and if a stable floating body be placed in the water it will carry its mast at right angles to the water, whether it owes its stability to deeply-stored ballast, or to a broad plane of flotation."

Now there are other conditions that must be complied with before the floating body can be swung in "any direction whatever;" and it is not impossible to spill the water "so long, merely, as the suspending lines are kept tight by the operator," for the water will inevitably be spilled if the motion be not communicated concurrently, slowly, and equably to the containing vessel and the water: moreover, when the model ship is placed on the mimic sea, it also must be so placed before motion is communicated to the dish and water, for if all are not put in motion at the same time, then the motion of the ship and water will not synchronize, for they will be subject to different amounts of force, and "the fundamental proposition" will be seen not to hold good.

The principles contained in the experiment are the same as those which obtain in the milkmaid's trick of swinging her pail of milk over her head, and in the practice of the sailor in swinging his sounding lead over his head: in each case sufficient velocity, in rotation, must be given to overcome the gravitating force, or the milk will be spilled, or the sailor will be advised of his stupidity by a good thump. In the case before us, unhappily, it is the sailor who is punished, because another, the philosopher, ignores the existence of a well-known law.

Mr. Froude, having stated his fundamental proposition, that "the surface of a fluid, when dynamically inclined, is virtually level to a body which floats on it, overlooks the essential difference between the conditions which obtain in his illustration, and those which obtain in a ship in a seaway.

Because the floating body on his mimic sea is subject to the like dynamic forces with the water, and moves with it, he assumes that a ship in a seaway "is subjected to the same dynamic force as the wave on which she floats, and might be treated as a surface particle on it," which is simply impossible, as a ship is subject to other forces than those which the sea give out; and though she is subject to the motions of the wave, she does not accept the wave motion, for if she did, her motions would always synchronize with those of the wave, which they never can entirely do.

And notwithstanding that the floating body on his mimic sea would continue to move with the water (if once set in motion with the water, and is not subject to any other force, as a ship amongst waves is), whether the form of body were V- or U-shaped, whether the centre of gravity were high or low, whether the body had one keel or a dozen, small or large; and this, because there is no motion of the particles of water amongst themselves, nor of the floating body amongst them, other than those produced by the one power; they *all* being moved by the same power at the same time, and to the due extent, according to their relative positions in the orbit of motion, the upper parts or particles moving in the least arcs, though their motions may be modified by the law of gravitation; yet every one of the above changes in form, in weight, and in disposition of the weights, would affect the motions of a ship in a seaway in facilitating or in retarding motion, so that a ship's motions could not synchronize with those of the wave. Moreover, as the motions of the water, in his illustration, are different from those of water in wave motion, in which the upper parts or particles moving with greater velocity than the lower, there is no analogy to the conditions of the water in the dish; also as the parts of a ship are substantially rigid, and one part occupying space amongst the fast moving particles, and another part amongst the slow moving particles, it is impossible that the ship could move wholly with either one set, or the other, or with both more than in part, and in some varying degree. Again, while in his illustration the containing vessel, the sea, the floating body, and *all* the water are moving together, under the impulse of the centrifugal force; in the case of a ship at sea there is a portion of every ship that is down amongst the unagitated water, *below* wave motion; this portion of the body would resist and retard motion, and prevent a ship from accepting and moving with the waves, if otherwise free or disposed to retain her masts perpendicular, to the slope of the wave. Indeed, were it true we should not have any such cases, as ships and boats being rolled over by waves, which are only too numerous, and will be more so in proportion as his ideas on that subject are adopted.

In fact in no case are the circumstances of a ship amongst waves similar to those of the illustrative model, nor can a ship ever "accept the aggregate dynamic conditions of the sea on which she floats," nor still less "be treated" with any regard to truth "as a surface particle of the wave on which she floats."

Indeed Mr. Froude's own scheme of a number of deep bilge-keels, for the purpose of checking motion, negatives the idea that a ship accepts the aggregate dynamic conditions of the sea, for it proceeds on the supposition that she does not do so, but, the contrary, that she rotates in, and through the wave, and not as "moving with the wave, and as virtually forming a part of it," so also is his panacea for rolling, contrary to the idea, that "the surface of the fluid when dynamically inclined, is virtually level to a body which floats on it" for therein is provision made for arresting the body by contact with the water.

There are also further limitations on the motions of a ship in a seaway, to which the model in Mr. Froude's illustration is *not* subject.

Thus the amount of a ships's motions are the result of the wind on the sails, or on the hull and masts, or the inertia of the masts and sides of the hull, together with the ever-varying resistances from onward, and from rotatory motion, while in the case of the model there is but one motor, and one influence ruling, that and gravity being common to both.

Nor can it be conceived with any approach to correctness that the place of the operator's hand, as specified in Mr. Froude's illustration, "holding ever so tightly the strings," can represent the *point* of buoyant suspension of a ship, or even on the principle that her motions are analogous, as they are not, to those of a conical or other pendulum, nor that the three strings represent the lines of buoyant power meeting in the hand of the operator.

These forces, as the ship rolls or is inclined, are continually changing in direction, and in amount, as it were, first on one string, then on the other; therefore the point of suspension and length of strings are continually changing.

Lastly and conclusively, the conditions of Mr. Froude's illustration require that the ship shall be suspended by three or more strings, or something analogous, and not that only, but that the wave or waves on which she partially and temporarily floats shall also be like as in the tray experiment suspended, nct that alone, but that each wave as it arrives up to, and passes under her shall be likewise so suspended; nor that only, but that the ship, the sea, even to the depth to which any portion of the ship is immersed, and a portion of which is always below the limit of the depth of wave motion, and all, altogether, shall be supported on three or more strings, and be all moved together, and to the same extent, by an invisible hand communicating motion from one point *above* the ship: whereas everyone must know, if he knows anything about the motions of ships at sea produced by waves alone, that instead of this imaginary hand, or power moving the ship from *above*, the ship is always in a seaway moved from *below*, without the intervention of strings, and that the *water* and not strings or a hand or anything analogous, is the motor, and which, unlike the hand and strings in the experiment, *never* succeeds in *wholly* imparting its own motion to the vessel floating on it.

There is nothing of that which is required by Mr. Froude's theory, which is therefore a mere fiction; and though there doubtless is a superstructure of superior mathematics built up by him, yet as it is *baseless*, his conclusions could not be otherwise than erroneous, and proportionably deceptive, and the adoption of them could only lead to the damage of our ships, and danger to the lives, if not also to the death of some of our seamen; and so it has proved, in the sad fate of the loss of the 'Captain,' and in the great danger to which other ships were exposed, as the writings of the advocates of these erroneous views have practically admitted.

Then he says, "The effect of stability is the lever by which a wave forces a ship into motion. If a ship were destitute of this stability no wave that the ocean produces would serve to *put* her in motion, *whether* the stability be due to deeply-stored *ballast* or to the broad plane of flotation."

Yet the effort of stability is originated by, and is *not* the, originator of, motion; it is the effort put forth in resisting motion and increases in amount with each increase of the extent of the motion, up to a certain point.

Stability may be obtained either by deeply-stored ballast, or by a broad plane of flotation, or in part by both; but they are unlike, and in some degree opposite in their operation and effects, yet here also Mr. Froude treats them as though they were strictly alike in their operation and in their effects.

No doubt the plane of flotation in proportion as it is wide when the sea is motionless, therefore not a motor, gives a very high degree of stability; but when the water is moved into waves, that same broad plane becomes proportionally *an instrument*, but not a motor, by which the waves give much greater motion to the ship, *they* being the power that puts the ship in motion—making her unsteady, yet not unstable, as is argued by Mr. Froude, for, in that case she would capsize, as his model also would, if unstable, and would not preserve its masts perpendicular to the surface of the water, this new hypothesis notwithstanding.

Unquestionably if a ship be without stability, as Mr. Froude suggests, she will be unstable, and not steady, but will do the opposite to that stated by that gentleman, *viz.* "not be rolled by any sea whatever," for she will certainly roll over, as was near being demonstrated in the case of the "Vanguard" and her sister ships; no doubt she will not roll in the sense of rolling from side to side, for being once rolled, and not possessing stability to bring her back to the perpendicular, even when the rolling force ceases to act, she will roll over.

Still less is there any tendency in ballast or a low centre of gravity "to *put* a ship in motion;" its action is to preserve the motionless condition of the ship, and it possesses no leverage till, by motion, it has been pushed out of the perpendicular by some force which is neither that of stability nor of ballast, ballast itself never being even like the broad plane of flotation, an instrument in originating greater motion; on the contrary, when it is pushed out of the perpendicular, its effort is always to bring the ship back to the motionless condition, and its effort in this direction is greater in proportion to the extent of the disturbance from the perpendicular, a disturbance originated by that other force; and, in proportion as the ballast or centre of gravity is lower, does it tend to limit the extent of the motion, which is the opposite of the action of a broad plane of flotation.

In proportion as the stability is derived from the weights being placed low, and relatively great as compared with the *smallness* of the plane of flotation, a ship will roll less and less.

Many have seen this fully illustrated in the steadiness or freedom from rolling with which a boathook floats in the midst of waves.

This condition is also well illustrated by a floating target, which remains vertical because of its studiously low-placed centre of gravity and small plane of floatation.

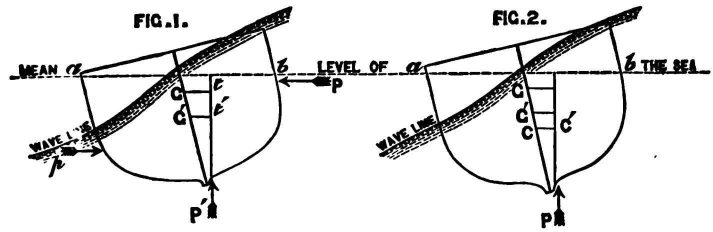
Let anyone take a boathook and load it still further near the hook, so as to push it well down into the water, and arrange that the stave or wood handle shall be as small as will consist with floating the added weight, if placed in the midst of high waves, it will be found to float, and continue to do so, nearly upright, the waves, as they pass, running up the stave, without inclining it further or rolling it, and why?

Because the great weight acting low down with all the force of the stability which it gives out, in consequence even of the small incline from the perpendicular, is exercised, not as Mr. Froude suggests, "to put the ship or handle in motion," but to prevent further motion except in the vertical plane; and the ballast succeeds in preventing motion, because the whole plane of flotation, or more properly the volume subject to immersion and emersion as the waves, and hollows, between the waves, pass, and which alone constitutes the instrument through which the waves act to *put* the ship in motion, is little or nothing compared to the force of the low weight, and to the action of the body low down in the undisturbed water, *resisting* motion. The obvious course therefore to be pursued, when it is desired to reduce the amount and the rapidity of motion in ships, would be to keep the plane of flotation comparatively small, the centre of gravity low, and make the depth half the breadth or as great as would consist with general good qualities, and the object for which any particular ship might be destined.

Instead of which two other courses are recommended by Mr. Froude and his school, viz., to raise the centre of gravity as compared with that in ships generally, and to distribute the weight outwards and on to the sides.

We may examine the effect of such recommendations with advantage.

The condition of ships in a seaway is misunderstood, as is the effect of increasing the inertia of their sides; the condition is not as supposed by Mr. Froude, that of a ship moving with, and as a particle in the wave; when waves are large or are steep, and are travelling fast across a ship's path, she has no time and may have very little tendency to accommodate her seat in the water to the surface or level of the wave, so that it becomes oblique to the ship's water-line, as in Fig. 1, producing an inequality in the pressures, which are the cause of the rolling motions. All the conditions of the pressures on the inclined body in still water as shown pages 73–4 of our ironclads and merchant ships, holds good equally as to the upright body with reference to an inclined surface of the wave.



In comparing two ships, or the same under different conditions, we must assume the wave to be alike in all particulars, then consider the variables and their consequences.

The disturbing wave force then is the same, whatever may be the form or distribution of weight in the ship, yet the effect of that force will vary considerably as the above are changed.

We will suppose Fig. 1 to represent the cross section of a ship, which, by weighting her sides, is prevented from inclining with the wave, or from accommodating herself as much as she otherwise would to the surface of the wave, which is represented by the wave line curve.

There is a complete change in the amount of the pressures from those that existed when the sea was level, and the ship upright; suppose the new pressures to be represented by the arrows at P, P', and p.

The water has accumulated at P, raising the point and increasing the amount of lateral pressure on that side, while the water has partially left the other side, decreasing the pressure there and lowering the point to p. These two pressures act as a couple to rotate the ship; that at P high up, and more high as the wave is higher, and its upper particles moving with greater velocity. That at p low and more low as the hollow of the wave is deeper, and their power to rotate the ship increases in the same extent.

Then P', moved over to the side of greater pressure, acts up on a line normal to the earth, and perpendicular to the mean level of the sea through t; G in the first instance representing the position of the centre of gravity, and $G\ t$ the perpendicular distance from the line of pressure; this also tends to turn the ship in the same same direction as the other pressures.

It is clear that the greater is the accumulation of water the further P' will be carried over, and the greater will be the lever $G\ t$, which in this case tends to overset; consequently any arrangements that would tend to increase this accumulation of water would be injurious and might be dangerous.

Now suppose the centre of gravity lowered to G'_1, then the upsetting lever will be $G'\ t'$, which is obviously shorter than $G\ t$; consequently, lowering the centre of gravity reduces the power of the wave to incline or roll the ship, and tends to limit the arcs rolled through.

Or suppose, though it is less accurate, these three forces to be represented by one at P passing up through the centre of figure C'. In Fig. 2, it is clear that the greater is the accumulation the greater distance C' will be moved out from a line passing through the middle of the section or ship, therefore the greater will be the distances of a perpendicular to the earth passing through C' from the centres of gravity at G or at G': obviously as in the former case, in proportion as the centre of gravity G is lowered, so the disturbing lever is decreased.

It is clear also that the greater is the breadth the greater distance will C' be carried out, and the greater will be the disturbing lever.

The greater is the accumulation of water also the further is C' carried out and the greater is the disturbing lever. The greater is the effect of Mr. Froude's recommendations, the greater becomes the upsetting lever.

The higher also the centre of gravity is, the longer is the disturbing lever and the greater is the extent of the motions and danger of upsetting. (See page 128 for a more definite proof.)

The more rapidly the wave is moving the greater is the difference between the motions of the upper and lower particles, therefore the higher up is the lateral thrust and the danger greater from a high centre of gravity.

It may be said that in lowering the centre of gravity the point round which the ship turns will be changed and, therefore, C' will not occupy the same place under the altered conditions. This is true, but the fact is in favour of our argument, for in proportion as the centre of gravity is low the radius will be shorter, C' therefore will be moved out a less distance and the injurious influence of the wave motion will be proportionably less.

To assume, therefore, that the metacentric height is a measure of the disturbing force is erroneous.

Is is indispensable for a ship's safety, as it also is for her complete efficiency, that she should possess a considerable metacentric height, then to reduce this by raising the centre of gravity under an impression that the arcs of roll will be reduced is erroneous and an unmixed evil.

It is not true, as stated by Mr. Froude, that "the effort of stability is the lever by which a wave forces a ship into motion. If a ship were destitute of this stability, no wave that the ocean produces would serve to put her in motion, whether the stability be due *to deeply stored ballast or* to a broad plane of flotation."

If, however, the metacentric height is made unduly high by great proportionate breadth or by placing a cargo of metal on the ship's floor, then the motions will be too *rapid* though short, in which case the evil may be got rid of, with benefit and without danger, by raising the centre of gravity. But still there is a minimum of metacentric height, below which it cannot be reduced with safety.

Ships must yield to the waves, and all arrangements and designs should be to facilitate their doing so slowly and equably. The effect of Mr. Froude's plan is to put off the inclination till the wave accumulates in such force that it will sweep the decks, force the ship over suddenly, and possibly overturn her, if she be not previously "swamped."

Therefore the metacentric height, or the distance of the centre of gravity from the metacentre, must not be taken as giving the measure of the disturbing force, for in lowering the centre of gravity the metacentric height is increased, but we have seen that in proportion as the centre of gravity is lowered so the disturbing action and extent of motion is limited; and therefore the proposition to raise the centre of gravity with a view to reduce a ship's motions is erroneous, and in proportion as it is raised it is dangerous.

No doubt the other proposition, that of increasing the moments of inertia by distributing the weights laterally, if it gives the wave, so to speak, more to do to lift this weight, on the one side;

it must be remembered that there is besides an accumulation of water on that side which overcomes these moments, but there is a withdrawal of water support from the other side, and that side tends to fall in search of support, and the more the weights are extended out on that side the greater is the force from that cause to rotate the ship.

Obviously their lateral distribution of weight increases and renders a ship's motions in a seaway dangerous in proportion as the waves are larger and more steep, the cure for which is concentration of weight laterally and distribution of weight downwards.

This has been immemorially the wise practice in bad weather, to send small yards and masts on deck, to fill empty tanks, and get the guns well in from the sides, and notoriously the ships rolled less and were easier, and yet the stability was increased by two causes, viz., lowering the centre of gravity and increasing the surface stability by reducing the weight of the sides, increasing their buoyant power. Consequently the only distribution of weight that is safe in bad weather is that which is downwards, and therefore raising weights off the bottom and concentrating them vertically is wrong on their own principle, as that reduces the inertia, and doubly wrong on the principle we contend for, viz., the propriety of lowering the centre of gravity and distributing the weights vertically.

The proposition to get rid of the difficulties arising out of the adoption of a wrong principle by increasing the breadth and endeavouring to limit the motions this will entail, by deep keels, will be a failure:—

First, it will not effect the object intended. We have passed the limit of practicable depth for useful ships, and any increase of breadth will increase the disparity between depth and half breadth, and occasion, as the ship rolls over and rolls back, a greater rise and fall of the centre of gravity, this action will occasion, so to speak, rolls, independent of those produced directly by the wave by which the total arc of roll will be increased, together with an increase of rapidity of roll, which keels cannot prevent if they will mitigate.

This rise and fall it was that caused the frightful rolling of Sir William Symond's ships.

What will happen in a ship with weight concentrated on her sides, high centre, and low freeboard, say on the L'Aghulas's bank, with a weather current and steep waves, will be that the great inertia of her sides will resist their rising to the wave, but finally, it will lift her suddenly, but a large body of water will break on board, and will rush over to the other side. She will then have three forces tending to turn her, the beam sea lifting the upper side, the weight of water weighing down the lower side, and the current running to windward below, making an adverse couple to roll her over, all which will be facilitated by the empty cells in her bottom. If she is not rolled over, it will be that a good Providence has intervened; if she does capsize, we shall know who to hang.

The same danger will arise from a weather tide or current, and from great tidal and solitary waves.

There are two other elements that materially affect the rolling motion, viz., length and depth, the greater these are, the greater is the resistance to rotatory motion Then, in proportion to the height of the waves, the depth of agitated water increases, and so does the portion of the ship down in unagitated water decrease, and with it the limitation to great motion decreases.

It is universally admitted that the lower the centre of gravity is, when once the ship is inclined by a force, the more rapidly will this low centre bring the ship back to the perpendicular, but then also, in contradiction to Mr. Froude's view must its power, of necessity, be greater to resist the force of the waves, acting through the solids of immersion and emersion to rotate the ship: also the wider the plane of flotation the greater will be the leverage and power of the sea to put a ship into greater motion: and, *cœteris paribus*, the higher the centre of gravity the less will be the leverage and power of the ballast, or low centre of gravity to resist motion. Therefore it is that in proportion as the centre of gravity is high, and the vessel broad, the effect of the sea in rolling the ship both rapidly and through large arcs is greater! But obviously it is not desirable that a ship should roll through large arcs; for in that case the armour, to be a real protection, must cover the whole side thus exposed from time to time at each roll, while, in addition, to a greater degree would the unarmoured deck be presented to the enemy, and the employment of guns on such a deck would be made proportionably difficult; still less is it desirable that these large arcs should be performed rapidly. Therefore the plane of flotation should not be greater than is necessary to give the requisite stability without having the centre of gravity too low.

The correctness of the above reasoning was abundantly shown in the behaviour of the ships forming the Channel Squadron in 1871, the large ironclads exhibiting to great disadvantage, when compared with the small wooden and unarmoured ships, though it is proverbial that, *cœteris paribus*, the larger the ship, the better the weather she ought to make.

Some of the ironclads rolled as much as 62°, some 56°, others 50°, while the much smaller "Topaze" rolled but 22°.

Again, while the little "Topaze" rolled but 22°, the large and most approved ironclad "Minotaur" rolled 39°, and the large and fine "Northumberland" rolled 38° under similar circumstances.

True, Mr. Reed attributes this difference to the existence of that which he thus admits, though he had previously denied it in former examples, the fact that the "Topaze" had larger masts and yards for her size than the others.

But we may ask if large masts produce such remarkably favourable, and, we may add, essential qualities, why not increase the size of the masts of the "Minotaur," as they would serve other useful purposes in addition? and why have shortened the masts in the "Vanguard" class?

The fact is that the effect of the difference of the size of the masts in mitigating the rolling motions of the "Topaze," was little compared to the disturbing effect of her broad plane of flotation, and but for which she would be easier in fine weather also, as she is so much easier in rough weather than the ironclads. The proportion of breadth to length in the "Topaze" was 1 to 4·7, while in the "Minotaur" it was 1 to 6·7; the proportion of breadth to length of the "Lord Clyde," which was the vessel that rolled through the largest arcs, was also 1 to 4·7. In fact the metacentric height of the "Topaze" was greater than that of most of the ships present on that occasion.

The rolling of these ironclads is without parallel amongst wooden vessels, and is only approached by those of the very worst form, and yet the ironclads are, in some respects, of a better form than the wooden ships; the excessive rolling, taking them as a whole, being mainly, though not entirely, due to the concentration of weight on the sides and want of stability, the fact being that no ironclad possesses near the amount of stability assigned by the ordinary mode of calculating; this arises from the excessive overloading of the solids of immersion and emersion, by the armour and by the other concentration of weight toward the sides, the consequence of which is that these portions of the ship are deprived of buoyancy, and they afford the ship little or no support, so when she is inclined they tend little or nothing, compared to their volume, to move the centre of buoyancy over to one side or the other, in rolling, so for want of this support amongst others, the ship rolls through large arcs, once the sea has overcome the inertia of the sides.

And this evil would increase in each, as they were lightened by the consumption of their coals, provisions, and water, and their stability thus reduced. Great as were the arcs of roll recorded of these ships, the probability is that with the exception of the "Lord Clyde" or another, very little of their weights were out; so they were in their best condition!—how bad the best!

The loss of the steam-ship "Tacna" is an instructive comment on the foregoing arguments, and a complete condemnation of the latest of the unsafe theories, *i.e.*, *that* "*a vessel whose stability is greatest when placed bottom upwards, may yet be perfectly safe when floating upright.*"

The description of the loss founded on a "mature consideration," as given by the Court, is "the loss of the ship 'Tacna' was due to an excessive loading of her main and hurricane decks, which with the combination of circumstances adduced in evidence as having arisen at the time of altering course for the port of Los Vilos, caused her to heel over until, falling on her beam-ends, she filled and foundered."

The Court condemned the Captain for "not having made known, in a more especial manner to his employers, the *crankness* of his vessel, and for not having exercised sufficient care in the amount of deck cargo."

Here is a ship "floating upright" till she comes to turn, when the centrifugal force, which is always greater as the centre of gravity is higher, capsized her. But will any reasonable man say she was safe?

Even a Pacific sea or a squall would have done equally what the centrifugal force did.

We have no question here as to a deficiency in the reserve of stability which a high side is said to give. If we ask where was her *reserve* of stability that she so went over, echo answers where?

The Captain was justly condemned for loading the ship as he did, knowing her to be crank; but he was not responsible for the latter, and he might in some degree be excused, when persons who are said to be the greatest authorities on such subjects say that crankness is a benefit and not a defect, so they have recommended little stability!

The ship was lost, like the "Captain," because "she was not endowed with sufficient initial stability;" for this the naval architect was primarily to blame, and merited condemnation by the Court as having contributed to the loss of life which occurred. Had the ship been lost within Chilian jurisdiction, it appears the Captain also would have lost his life.

The ship never was seaworthy from deficiency in her initial stability, and whether that proceeded from direct design, from a double bottom, and from not making due allowance for that, or from an error in calculation, the naval architect alone was to blame and ought to be held responsible, and it will be useless legislating for the safety of life and property if naval architects are allowed to do as seems good in their own eyes without being held accountable. To say that the deck load capsized this vessel is no exoneration, as the Froude-Reed system provides that ships in a seaway shall have a deck load of the greatest, most generally damaging and dangerous kind, that of water, which will rush to the lower side and guarantee an upset, for in providing, so far as they could, that a ship shall remain upright in a sea, they provide that the waves shall roll into and over her.

These gentlemen may say, Oh, no, we think there ought to be sufficient stability to enable ships to carry the amount of sail with which they are furnished. Let us consider this:—

The maximum pressure of wind must be assumed, acting at the given leverage occasioned by height and area of sail, in fact the stability must be equal to the force shewn by the wind couple in the case, and more, as a force coming suddenly is said to occasion double the inclination, the stability should be double this; then the stability should be sufficient to bear the ship up against inevitable deck loads of water, or a blow of a heavy sea striking her when she is inclining, such as proved fatal to the "Captain," because of her small initial stability.

The metacentric height, while that is the accepted mode of estimating stability, ought not to be less than six feet for smaller, and five feet for larger sailing vessels, somewhat less for steam vessels with little more than fore and aft sail.

It is clear no such quantity was contemplated for the "Invincible," or "Sultan," and a fleet of other men of war, which a deck load of water from a sea, or other weight would have cut short in their career while hardly begun, if they had not been heavily ballasted, and yet a vessel of war shorn of the power to carry deck loads would often be useless; thus on an 800 ton steamer it depended to keep up communications with the army in Kaffraria, and the colony, to throw in reinforcements, provisions, horses and munitions of war, carrying sometimes as many as 800 levies with their stores and provisions, and this all on deck. Yet this vessel was proverbially an easy vessel. Numberless instances of the necessity for such a property could be given.

It may be affirmed also with confidence that various exigencies occur, when such a power is necessary in merchant ships, such as taking crews off sinking vessels, and yet no such power was provided for in the many ships that have capsized because of their having deck loads.

It is worthy of observation when considering any other enactment, that Mr. Plimsoll's proposed Act, if in force, would not have saved ships in the condition of the "Tacna;" she was lost in summer, in the Pacific Sea, and was or might not have been immersed boyond his proposed maximum line.

Naval architects and shipowners should be held responsible, the former especially in such cases, and their conduct should be the subject of animadversion by the *Court*, equally with that of the captain, officers, and seamen, and if the law does not enforce a minimum measure of stability at loadline, with an assumed centre of gravity, which should be recorded as an element in safety, in case of loss a deficiency in that respect should be a ground of charge against the naval architect, and his name should be gazetted, if he were not otherwise punished.

It may be said that we object *in toto* to double bottoms, though they are necessary, because the thin iron skin is more easily perforated than is the wood planking of wood ships.

We do not object to any proper use of a second or inner bottom, to make iron ships stronger and safer against injuries from taking the ground or from torpedoes, or to facilitate repairs:

such ships require it, as their bottoms are far less strong and less calculated to bear injury than those of wood ships; and we doubt not those iron vessels, with deep iron bilge-keels, will be specially liable to damage; nay, more, that these very keels, on taking the ground, in many cases will serve to tear a large portion of the lower bottom out, if they do not injure the inner or upper bottom, through the frames, thanks to the little-stability system, smuggled into the Navy to "*check* rolling."

We do, however, object to raising these inner bottoms unduly, for the purpose of raising the centre of gravity, from an idea that it is wise to do so.

We object to making the bottoms of vessels light, on the grounds of cheeseparing parsimony, and then assuming that this makes no difference in the properties of the vessel, that the bottoms are equally heavy, though empty, with all other parts of the immersed body, even when they are crammed full, and then assuming that the results of calculations made on such an hypothesis can be otherwise than erroneous, deceptive, and proportionably dangerous. That they are so we have proved in pages 34 to 44 of "Our Ironclads and Merchant Vessels," and need not repeat it here.

We have also demonstrated that in proportion as the spaces between the bottoms are large and empty it will be necessary for safety to lower the centre of gravity, it will be necessary, also, with the same object, to lower the centre of gravity in proportion as the sides are more and more loaded and deprived of buoyant power.

The oscillations of wide vessels with heavily weighted sides are liable to carry them beyond the upsetting angle; for this reason, also, their centres of gravity would require to be kept lower.

It will be asked what, was there nothing in past experience to justify the adoption of the Froude-Reed system? No facts, no experiments, proving its correctness and safety? absolutely none.

The one complaint in which all were for long agreed was, that our ships were deficient in stability from which they always suffered in comparison with French and Spanish ships. This led to the introduction of Sir William Symond's system, and the success of his ships was entirely due to their greater stability, which they maintained till greater stability was given by his opponents.

His system was exploded because his stability was, for the most part, obtained by a broad plane of flotation, therefore his ships were more at the mercy of the waves, and their motions were extravagantly extensive; the more so that they were without a low centre of gravity to limit the range of motion.

When steam was introduced the engines and boilers took the place of ballast in keeping the centre of gravity low, more necessary because of the reduced breadth to obtain speed economically, and necessary because of the great reduction of stability when the latter part of the coals were being consumed.

A lower centre of gravity is necessitated in merchant vessels by the fact that many of them are obliged to carry deck loads.

As great stability and much of it by a low centre of gravity was adopted as indispensable by common consent alike of sailors and naval architects, the result of long extensive and conclusive experiments and represented by a metacentric height of five feet to six feet or more, an architect giving much less to a design would have been justly esteemed as foolish or criminally ignorant. The laws of nature being unchanged, how can it be otherwise now in giving metacentric heights such as 2·6 feet, 2·3 feet, 2·2 feet, and even 1·5 feet under other much more unfavourable conditions? There was nothing in the facts offered in justification of the course pursued.

As we now propose to show;—Mr. Reed wrote, "the steadi-"ness at sea of our ironclads is due to their want of stiffness or "stability:" also that it had been found that raising the centre gravity tended to "check rolling, and that it was a popular fallacy to suppose the armour-made ships roll."

It was recommended by Mr. Froude with a view to reduce the tendency to roll to raise the centre of gravity of ships, and to distribute their weights towards their sides.

Sir Spencer Robinson accepted the principle involved in these statements and writes:—"It is remarkable that, according to theory, the rolling of the ships being very much influenced by the position of the centre of gravity with regard to the metacentre, and by the moment of stability, the order of rolling of the French ships as observed, follows that law."

The height of the metacentre above the centre of gravity in these ships is as follows:—

Armoured	"Solferino," 4·5 ft.,	armoured	"Couronne,"	5·37 ft.
Unarmoured	"Napoleon," 4·9	„	"Invincible,"	6·36 ft.
Armoured	"Magenta," 5·0	„	"Normandie,"	6·59 ft.
Unarmoured	"Tourville," 5·31 ft.			

"This," he says, is "precisely the order of merit they have taken as to rolling." The "Solferino" rolling least and the "Normandie" the most.'

Now we unhesitatingly affirm that there is nothing to justify the conclusion that these gentlemen have come to, viz., that the greater rolling was caused by the greater metacentric height, but the reverse.

We accept the statement as to the performances of these ships as given by Sir. S. Robinson.

1st. He says "that the 'Solferino' is superior to all the ships of both squadrons," but this includes the "Achilles," yet her metacentric height is only 3·1 feet, while that of the "Solferino" is 4·5 feet, therefore the theory breaks down, nor is this all, for the "Solferino" is nearly one third smaller, *i.e.*, of 2,600 tons less displacement than "Achilles," and, therefore, might reasonably be expected to roll more instead of less, and while they have the same extreme breadth, which is the disturbing dimension, the "Achilles" is 90 feet longer!

The decided superiority of "Solferino" must be due to the lower centre of gravity and greater metacentric height, and the weights being on her bottom, tending to limit the arcs rolled through, or to "check rolling."

Again he states "that 'Achilles' is quite as good as the 'Napoleon,'" what? not much better! Why the metacentric height of "Napoleon" is 4·9 feet, that of the "Achilles" only 3·1 feet! The disturbing force of "Napoleon" may be represented by the cube of 55, that of the "Achilles" by the cube of 58, but then the latter is 147 feet longer and has only 4,400 tons greater displacement! Moreover, the French ship not being an ironclad did not possess the alleged soporific of weighted sides. The theory here is doubly in fault. The true explanation is a low centre of gravity, great metacentric height, and weights on the floor.

Then the "Achilles" is said to be rather superior to the "Magenta," when she ought to be very superior, being so much larger, that is, 90 feet longer and 2,600 tons more displacement,

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Then the "Achilles" is said to be rather superior to the "Magenta," when she ought to be very superior, being so much larger, that is, 90 feet longer and 2,600 tons more displacement,

with same breadth of beam, *i.e.*, an equal disturbing element. Here, also, the theory fails, and the explanation is, a lower centre of gravity and weights on the floor!

Also, "Achilles" is stated to be decidedly superior to the rest of the French squadron, excepting the "Tourville." Why, the metacentric height of this ship is 5·3 feet, that of "Achilles" 3·1 feet, she is without the soporific of armour, is 140 feet shorter, and 4,000 tons less displacement than "Achilles." Again the theory is doubly wrong.

"Achilles" is superior to the rest of the French squadron, *i.e.*, to the "Couronne," the "Invincible," and the "Normandie." Really we are surprised that a direct comparison should have been made; she is 110 feet longer than "Couronne," and 3,300 tons more displacement, and 118 feet longer than each of the other two, and 3,600 tons greater displacement than one, and 3,890 tons more than the other. That these did not roll very much more than her is due to their higher centre of gravity and greater metacentric height.

For, comparing the French ships amongst themselves in respect of their metacentric height, there is not even the show of reason, for, 1st, the two unarmoured ships must be struck out as it is now admitted that they roll less than ironclads. The theory as to the contrary is now abandoned by Mr. Reed, reason and experience have re-obtained their sway.

The French ironclads take their places in the order of their size, the largest rolling least. The two first are sister ships as also the two last, but of a lower class, the proportion of *breadth* to length in these last is greater than that of the other three, therefore they ought to roll more. The "Magenta" is 1,100 greater displacement than the "Normandie." Then the "Edgar"* is said to be inferior to the 'Napoleon,' 'Tourville,' 'Solferino,' 'Achilles,' and 'Magenta.'" The inferiority cannot arise from her metacentric height for it is equal to that of the "Napoleon" but less than that of all the other French ships. Why should she be inferior to the "Napoleon" and "Tourville?" because being three feet wider, she is subject to greater disturbance; the other ships are all so much larger, especially the "Achilles," that no comparison in reason can be made, the "Achilles" is 140 feet longer, and of 4,000 greater displacement.

* We take the metacentric height of "Edgar" to be the same as her sister ship, 4·6 feet.

We take the metacentric height of "Black Prince" to be the same as that of her sister ship "Achilies," that of "Hector" is 4·6, and "Black Prince" is a sister ship to "Achilles," and ought not, are far as we are informed, to be different from her.

The "Defence" is said to be better than the "Hector" and "Prince Consort." With the equal length the "Defence" has two feet less breadth than the "Hector," and with 7 feet greater length she has 4 feet less breadth than the "Prince Consort," therefore we should expect the latter to have greater motion than either of the others.

Then the "Black Prince," with her smaller metacentric height, 3·1 feet, is admitted to be inferior to the "Solferino" and "Magenta" with their considerable height of metacentre, one 4·5 the other 5 feet.

We have before examined the facts offered in support of the theory as drawn from the comparison of the rolling of a number of English ironclads, and have shown they range themselves as respects their rolling exactly in the order of the proportion of breadth to length, the vessel with least proportionate breadth rolling least. The "Minotaur," the largest and best ship with a metacentric height of 3·8 feet, superior to the "Bellerophon" with only 3·2 feet of metacentric height.

The "Pallas," a ship of Mr. E. J. Reed's, the worst roller, her smaller metacentric height not checking the rolling so much as the great metacentric height of the "Prince Consort," 6·1 feet.

It was natural for "Pallas," to roll she had such a great proportionate breadth of plane of flotation.

Raising weight did not cure, but the contrary it made her rolling worse.

Obviously the theory that induced these gentlemen to damage a fleet of ships by reducing their stabilities has no foundation in experience, in reason, or in science, and they ought to be held responsible for the loss of ships by capsizing and loss of life that arises from their improper advocacy of insufficient stability.

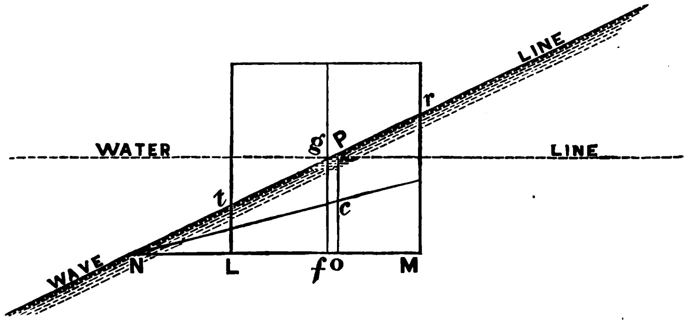
"'Tacna' to wit!"

And as this system might have been effectually tested in the course of an afternoon, not one of these unsafe ships should have been built.

We take the metacentric height of "Black Prince" to be nearly that of her sister ship "Achilles." That of "Hector" is 4·6 feet.

Proof of Proposition stated page 115.

If, instead of supposing the floating body to be inclined at any given angle in smooth water, we suppose the surface of the wave to be inclined to the mean surface at an equal angle, it will be evident that the reasoning with reference to the pressures, pages 73 and 74, will hold good.



Take $Mr = b$, $Lt = a$, $NM = c$ $NL = d$ $NO = X$.

Taking the usual formula for the centre of gravity, and integrating between the limits c and d.

$$X = \frac{\int xy\delta x}{\int y\delta x}$$

$$y = \frac{b}{c}x$$

$$\int xy\delta x = \frac{b}{c}\int x^2\delta x = \frac{b}{c}\frac{x^3}{3}$$

$$\int y\delta x = \frac{b}{c}\frac{x^2}{2}$$

$$\frac{\int_d^c xy\delta x}{\int_d^c y\delta x} = \frac{h}{3}\frac{c^3 - d^3}{c^2 - d^2} =$$

$$\frac{2}{3}\frac{c^2 + cd + d^2}{c + d}$$

$c + d = 24$ $\quad$ $3(c + d) = 72$

$c^2 + cd + d^2$

$324 + 108 + 36 = 468$

$2(c^2 + cd + d^2) = 936$

$$X = \frac{936}{72} = 13.$$

Formula for the equilibrated lever $\frac{b^3}{12S} - a.$

Centre of gravity of the system being above the centre of displacement.

In this formula, $b =$ the line of intèrsection of the cross section of the floating body when upright with the surface of the water.

S, the figure of displacement when the body is upright; a, the distance between the centres of gravity and displacement when the body is upright.

In this case, that of a block 12 feet square as described, page 72, &c.,

$$b = 12 \text{ and } S = 6 \times 12.$$

$$\frac{b^3}{12^2 \times 6} = \cdot 5$$

$$op = \frac{13}{2} = 6 \cdot 5.$$

$$oc = \frac{op}{2} = 3 \cdot 25.$$

g being the centre of gravity of the floating block $f\ g$, evidently $= 6$.

$$a = b - 325 = 275$$

$$\frac{b^3}{12S} - a = \cdot 5 - 2 \cdot 75 = -2 \cdot 25.$$

The stability being negative in this case; but as the centre of gravity is lowered, a is diminished till it becomes equal to $\frac{b^3}{12S}$, when the stability is that of indifference; when a is less than $\frac{b}{12S}$, the stability becomes positive and the righting lever increases, while the upsetting lever remains constant; the conditions in other respects remaining unaltered.

PROBLEM V.

To find the nature of the curves made by cutting the fore body by planes perpendicular and parallel to the upright central longitudinal plane.

Let $OXDZ$ be half the fore body of the ship, from the keel DZ to the greatest deck plane XYX_1Y_1, Fig. XXXIV, YZY_1Z_1; the ellipse which divides the fore from the aft body; and $mCPM$ a plane passing through the fore body parallel to the plane OYX. $OX = a$; $OX_1 = c$; $OY = OY_1 = 2b$; $OZ = OZ_1 = h$; $On = x$; $nN = z$; $NP = y$, P being any point on the surface. The plane $ApBq$, passing through P, is supposed to be perpendicular to the plane XqX_1p and parallel to the plane ZYZ_1Y_1.

$$np = nq = b\left(1 + \cos\frac{\pi x}{a}\right) = 2b\cos^2\frac{\pi x}{2a};$$

for, by trigonometry, $1 + \cos\theta = 2\cos^2\frac{\theta}{2}$, θ being the length of any circular arc to radius 1. Putting mM, which is supposed to be perpendicular to OZ, $= 2v$, we have,

$$Z_1O^2 : OY^2 :: Z_1m \times mZ : mM^2,$$

that is,

$$h^2 : 4b^2 :: h^2 - z^2 : 4v^2.$$

$$\therefore 2v = \frac{2b}{h}(h^2 - z^2)^{\frac{1}{2}}.$$

$$y = NP = 2v\cos^2\frac{\pi x}{2a} = \frac{2b}{h}(h^2 - z^2)^{\frac{1}{2}}\cos^2\frac{\pi x}{2a};\ mN = On = x.$$

It is evident that $qApPB$, any section parallel to YZY_1Z_1, is an ellipsis, for $An = nB = h$; $np = nq = 2b\cos^2\frac{\pi x}{2a}$, the principal axis of the section; $nN = z$, then, $An^2 : np^2 :: AN \times NB : NP^2$, that is,

$$h^2 : 4b^2\cos^4\frac{\pi x}{2a} :: (h^2 - z^2) : \frac{4b^2}{h^2}(h^2 - z^2)\cos^4\frac{\pi x}{2a};$$

but

$\frac{4b^2}{h^2}(h^2 - z^2)\cos^4\frac{\pi x}{2a}$ is equal to the square of $\frac{2b}{h}(h^2 - z^2)^{\frac{1}{2}}\cos^2\frac{\pi x}{2a}$.

consequently $ApPBq$ is an ellipsis.

The equation, to any point on the surface of the half fore body $YpXDZMY$, is

$$y=\frac{2b}{h}(h^2-z^2)^{\frac{1}{2}}\cos^2\frac{\pi x}{2a}=b\left(1+\cos\frac{\pi x}{a}\right)\frac{(h^2-z^2)^{\frac{1}{2}}}{h};\ (1).$$

To find the equation of the curve formed by cutting the fore body by a plane, passing through any point P, parallel to the vertical plane $OXDZ$. In equation (1), y becomes a constant quantity, which put $=f$; then (1) may be put under the form

$$fh=b\left(1+\cos\frac{\pi x}{a}\right)(h^2-z^2)^{\frac{1}{2}};\ (2).$$

$$\therefore f:b\left(1+\cos\frac{\pi x}{a}\right)::(h^2-z^2)^{\frac{1}{2}}:h.$$

With $np=b\left(1+\cos\frac{\pi x}{a}\right)$ = the ordinate directly over P on the greatest horizontal plane, describe a circle KpL, Fig. XXXV; take the perpendicular $Tp=f$, draw npS cutting the circle, described with nB as radius, in the point S; draw Sn perpendicular to $nB=h$; then $nN=z$.

$$(nS)^2-(nN)^2=(NS)^2,\text{ that is }(h^2-z^2)^{\frac{1}{2}}=NS.$$

By similar triangles we have, $Tp:pn::NS:Sn$, that is,

$$f:b\left(1+\cos\frac{\pi x}{a}\right)::(h^2-z^2)^{\frac{1}{2}}:h.$$

Therefore, for $On=x$, n being any point in $OX=a$; we find $nN=z$ by construction.

When $x=o$, equation (2) becomes $fh=2b\,(h^2-z^2)^{\frac{1}{2}}$, and $z=OR=h\left(1-\frac{f^2}{4b^2}\right)^{\frac{1}{2}}$. Now, if $\frac{f}{2b}$ = the sine of an arc ϕ, to radius 1, then, $z=OR=h\cos\phi$.

Again, if $z=o$, (2) becomes $f=b\left(1+\cos\frac{\pi x}{a}\right)$;

$$\therefore x=OQ=\frac{a}{\pi}\cos^{-1}\left(\frac{f}{b}-1\right);\text{ or }=\frac{2a}{\pi}\cos^{-1}\left(\frac{f}{2b}\right)^{\frac{1}{2}}.$$

Ex. Let XX_1, Fig. XXXIV, $=245$ ft.; $OX=147$ ft., $OX_1=98$ ft.; $OY=OY_1=2b=17{\cdot}5$ ft.; $OZ=15$ ft.$=h$; $On=x=56$ ft.; and $NP=f=5$ ft.

Then in Fig. XXXV, $OX=a=147$ ft,; $OZ=nB=h=15$ ft.;

taking n, any point in OX, we have, in the present case, taken On=56 ft.

$$\downarrow,\left(\frac{f}{2b}\right)^{\frac{1}{2}} = \frac{\downarrow,(f)-\downarrow,(2b)}{2} = \frac{\downarrow,(\cdot 5)-\downarrow,(1\cdot 75)}{2} = '62638149.$$

$$\begin{array}{rl} \downarrow,(\cdot 5) = & '69314718 \\ -\downarrow,(1\cdot 75) = & '55961579 \\ & 2)\overline{'125276297} \\ & '62638149 = \downarrow,(\cdot 534522482) \end{array}$$

$$\cos(57^\circ\ 41'\ 18''\cdot 45) = \cdot 5345225 = \sin(32^\circ\ 18'\ 41''\cdot 55).$$

Length of arc of 57° 41′ 18″ ·45, to radius 1, =1·0068535;

$$\therefore\ x = OQ = \frac{2a}{\pi}\cos^{-1}\left(\frac{f}{2b}\right)^{\frac{1}{2}}$$

$$\therefore\ \downarrow,(x) = \downarrow,(2a) - \downarrow,(\pi) + \downarrow,(1\cdot 0068535).$$

$$\begin{array}{lrr} \downarrow,(2a) = \downarrow,(2\cdot 94) = & 107840958,\ \text{ar. co.} & \bar{1}892159042 \\ \text{minus, } \downarrow,(\pi) = \downarrow,(3\cdot 14159265) = & '114472989 & '114472989 \\ \downarrow,(1\cdot 00685350) = & 683014,\ \text{ar. co.} & \underline{\bar{1}316986} \\ \downarrow,(\cdot 94224478) = & & '5949017 \end{array}$$

$$\therefore\ OQ = 94\cdot 224478 \text{ ft.}$$

$$OR = h\left(1-\frac{f^2}{4b^2}\right)^{\frac{1}{2}} = h\cos\phi.\quad \downarrow,\left(\frac{f}{2b}\right) = '125276297$$

$$\begin{array}{rl} & \underline{230258509}, = \downarrow,(10) \\ \downarrow,(2\cdot 85714283) = & 104982212, \end{array}$$

$$\therefore\ \frac{f}{2b} = \cdot 2857143 = \sin(16^\circ\ 36'\ 5''\cdot 59)$$

$$\cos\phi = \cos(16^\circ\ 36'\ 5''\cdot 59) = \cdot 9583149$$

$$\therefore\ OR = h\cos\phi = 15\times(\cdot 9583149) = 14\cdot 374724.$$

To find $nN=z$, when $On=x$ is taken = 56 ft.; equation (2) may be presented under the form

$$\frac{f}{2b\cos^2\dfrac{\pi x}{2a}} = \left(1-\frac{z^2}{h^2}\right)^{\frac{1}{2}},$$

in which $2b\cos^2\dfrac{\pi x}{2a}$ = the ordinate np, Fig. XXXIV.

$$\frac{\pi x}{2a} = \frac{\pi\times 56}{294} = \cdot 598398504,\ \text{since } \pi = 3\cdot 14159265;$$

now, ·5983985 is the length of an arc of 34° 17′ 10″·32 to radius 1; and sin (55° 42′ 49″·68) or cos (34° 17′ 10″·32 = ·8262339, the dual log., corresponding to which is = '19087739.

$$\therefore \downarrow, \left(\cos^2 \frac{\pi x}{2a}\right) = {}'38175478$$

$$\downarrow, (2b), \text{ or } \downarrow, (1{\cdot}75) = 55961579,$$

$$17786101, \ = \downarrow, (1{\cdot}19465926)$$

$$\text{and } \therefore np = 11{\cdot}9465926.$$

But, $\dfrac{f}{2b\left(\cos^2\left(\dfrac{\pi x}{2a}\right)\right.} = \left(1 - \dfrac{z^2}{h^2}\right)^{\frac{1}{2}}$ = the cosine of an arc of which $\dfrac{z}{h}$ is the sine.

$$\downarrow, (f) - \downarrow, \left(2b \cos^2 \frac{\pi x}{2a}\right) = \downarrow, (5{\cdot}) - \downarrow, (11{\cdot}9465926) =$$

$$\downarrow, ({\cdot}5) - \downarrow, (1{\cdot}19465926);$$

$$\downarrow, ({\cdot}5) = {}'69314718$$

$$\text{minus } \downarrow, (1{\cdot}19465926) = {}'17786101$$

$${\cdot}87100819 = \downarrow, ({\cdot}41852938);$$

hence, ·4185294 = cos (65° 15′ 29″·61), and sin (65° 15′ 29″ ·61) = ·9082033;

$$\therefore \frac{z}{h} = \frac{z}{15} = {\cdot}9082033, \text{ and,}$$

$$nN = 13{\cdot}62305 = ({\cdot}9082033) \times 15.$$

PROBLEM VI.

To find the nature of the curves made by cutting the aft body by planes perpendicular and parallel to the upright central longitudinal plane.

XYX_1Y_1, Fig. XXXVI, represents the horizontal and greatest deck plane; $OX_1 = c$; $OX = a$; $OY = 2b = OY_1$; ZYZ_1Y_1 is an ellipsis passing through the greatest breadth YOY_1 the plane of which is perpendicular to the plane XYX_1Y_1, $OZ = h = OZ_1$; and $mMPCNm$ represents a plane cutting the solid parallel to the plane XYX_1Y_1.

Let $BPpAq$ be a plane passing through any point n parallel

His system was exploded because his stability was, for the most part, obtained by a broad plane of flotation, therefore his ships were more at the mercy of the waves, and their motions were extravagantly extensive; the more so that they were without a low centre of gravity to limit the range of motion.

When steam was introduced the engines and boilers took the place of ballast in keeping the centre of gravity low, more necessary because of the reduced breadth to obtain speed economically, and necessary because of the great reduction of stability when the latter part of the coals were being consumed.

A lower centre of gravity is necessitated in merchant vessels by the fact that many of them are obliged to carry deck loads.

As great stability and much of it by a low centre of gravity was adopted as indispensable by common consent alike of sailors and naval architects, the result of long extensive and conclusive experiments and represented by a metacentric height of five feet to six feet or more, an architect giving much less to a design would have been justly esteemed as foolish or criminally ignorant. The laws of nature being unchanged, how can it be otherwise now in giving metacentric heights such as 2·6 feet, 2·3 feet, 2·2 feet, and even 1·5 feet under other much more unfavourable conditions? There was nothing in the facts offered in justification of the course pursued.

As we now propose to show;—Mr. Reed wrote, "the steadi-"ness at sea of our ironclads is due to their want of stiffness or "stability:" also that it had been found that raising the centre gravity tended to "check rolling, and that it was a popular fallacy to suppose the armour-made ships roll."

It was recommended by Mr. Froude with a view to reduce the tendency to roll to raise the centre of gravity of ships, and to distribute their weights towards their sides.

Sir Spencer Robinson accepted the principle involved in these statements and writes:—"It is remarkable that, according to theory, the rolling of the ships being very much influenced by the position of the centre of gravity with regard to the metacentre, and by the moment of stability, the order of rolling of the French ships as observed, follows that law."

The height of the metacentre above the centre of gravity in these ships is as follows:—

Armoured "Solferino," 4·5 ft., armoured "Couronne," 5·37 ft.
Unarmoured "Napoleon," 4·9 „ "Invincible," 6·36 ft.
Armoured "Magenta," 5·0 „ "Normandie," 6·59 ft.
Unarmoured "Tourville," 5·31 ft.

"This," he says, is "precisely the order of merit they have taken as to rolling." The "Solferino" rolling least and the "Normandie" the most.

Now we unhesitatingly affirm that there is nothing to justify the conclusion that these gentlemen have come to, viz., that the greater rolling was caused by the greater metacentric height, but the reverse.

We accept the statement as to the performances of these ships as given by Sir. S. Robinson.

1st. He says "that the 'Solferino' is superior to all the ships of both squadrons," but this includes the "Achilles," yet her metacentric height is only 3·1 feet, while that of the "Solferino" is 4·5 feet, therefore the theory breaks down, nor is this all, for the "Solferino" is nearly one third smaller, *i.e.*, of 2,600 tons less displacement than "Achilles," and, therefore, might reasonably be expected to roll more instead of less, and while they have the same extreme breadth, which is the disturbing dimension, the "Achilles" is 90 feet longer!

The decided superiority of "Solferino" must be due to the lower centre of gravity and greater metacentric height, and the weights being on her bottom, tending to limit the arcs rolled through, or to "check rolling."

Again he states "that 'Achilles' is quite as good as the 'Napoleon,'" what? not much better! Why the metacentric height of "Napoleon" is 4·9 feet, that of the "Achilles" only 3·1 feet! The disturbing force of "Napoleon" may be represented by the cube of 55, that of the "Achilles" by the cube of 58, but then the latter is 147 feet longer and has only 4,400 tons greater displacement! Moreover, the French ship not being an ironclad did not possess the alleged soporific of weighted sides. The theory here is doubly in fault. The true explanation is a low centre of gravity, great metacentric height, and weights on the floor.

Then the "Achilles" is said to be rather superior to the "Magenta," when she ought to be very superior, being so much larger, that is, 90 feet longer and 2,600 tons more displacement,

$$\therefore \quad u = \frac{2c}{\pi} \cos^{-1}\left(\frac{f}{2b}\right)^{\frac{1}{3}}.$$

We have before shown that $\left(\frac{f}{2b}\right)^{\frac{1}{3}} = \cdot 53452248 =$ the sine of $32^\circ\ 18'\ 41''\cdot 55 =$ the cosine of $57^\circ\ 41'\ 18''\cdot 45$; and $1\cdot 0068535$ length of arc of $57^\circ\ 41'\ 18''\cdot 45$;

$$\therefore \ \downarrow, (u) = \downarrow, (2c) - \downarrow, (\pi) + \downarrow, (1\cdot 0068535).$$

	$\downarrow, (2c), \downarrow, (1\cdot 96) =$	67294447,	ar. co. $\bar{1}32705553$
minus,	$\downarrow, (\pi) =$	$'114472989$	$'114472989$
	$\downarrow, (1\cdot 0068535) =$	683014,	ar. co. $\bar{1}316986$
		$'46495528$	$'46495528$

$$'46495128 \ \downarrow, (\cdot 628163201)$$

$$\therefore \quad u = 62\cdot 81632 \text{ ft.}$$

To find x when $z = o$, we have, $x = u + b \sin \frac{\pi u}{c}$;

to find $\frac{\pi u}{c}$,

	$\downarrow, (\pi)$	$= 114472989$,	114472989,
	$\downarrow, (u)$; $\downarrow, (\cdot 628163201)$	$= {}'46495528$ ar. co.	$\bar{1}53504472$
minus,	$\downarrow, (c)$; $\uparrow, (\cdot 98)$	$= 2020271$,	2020271
		69997732,	69997732'

$$'69997732 = \downarrow, (2\cdot 01370703).$$

$3\cdot 14159265 - 2\cdot 01370703 = 1\cdot 12788562 =$ an arc of $64^\circ\ 37'\ 23''\cdot 09$ to radius 1.

This result is correct, for

$$\frac{\pi u}{c} = \text{double} \frac{\pi u}{2c},$$

and $64^\circ\ 37'\ 23''\cdot 09 =$ twice ($32^\circ\ 18'\ 41''\cdot 55$ an arc, before found; and $2\cdot 0137070 =$ twice $1\cdot 0068535$.

$$\sin (64^\circ\ 37'\ 23''\cdot 09) = \cdot 903508;$$

$$b \sin \frac{\pi u}{c} = 15 \times (\cdot 903508) = 13\cdot 55262;$$

$$\therefore x = u + b \sin \frac{\pi u}{c} = 62\cdot 81632 + 13\cdot 55262 = 76\cdot 36894 \text{ ft.}$$

When $u = o$, $x = o$, and $2b \cos^2 \frac{\pi u}{2c}\left(1 - \frac{z^2}{h^2}\right)^{\frac{1}{2}} = f$, becomes

$$2b\left(1 - \frac{z^2}{h^2}\right)^{\frac{1}{2}} = f;$$

$$\therefore \left(1 - \frac{z^2}{h^2}\right)^{\frac{1}{2}} = \frac{f}{2b} = \frac{5\cdot}{17{\cdot}5} = \frac{2}{7} = {\cdot}28571428.$$

Now if ·2857143 be the sine of an angle to radius 1, $\frac{z}{h}$ will be the cosine of the same angle; but, $\cdot 2857143 = \sin (16^\circ\ 36'\ 5''{\cdot}59)$, and $\cos (16^\circ\ 36'\ 5''{\cdot}59) = {\cdot}9583149$;

$$\therefore z = h \times ({\cdot}9583149) = 14{\cdot}374724 = OR,$$

(Fig. XXXV), a result before found.

To construct the curve lde, Fig. XXXVII, formed by cutting the aft body of the ship by a plane parallel to the centre upright plane $OZDX_1$.

For any point n, in OX_1, let

$$Og = u;\ On = x = u + b \sin \frac{\pi u}{c};\ c = OX_1.$$

The ordinate np, Fig. XXXVI, on the aft main deck

$$= b\left(1 + \cos \frac{\pi u}{c}\right) = 2b \cos^2 \frac{\pi a}{2c};$$

and $nB = Oz = h$. With n as a centre and np, the main deck ordinate at n, describe the circle $q_1 m_1 p$; draw the radius npr through the point p where pN, perpendicular to nB, $= f$ the distance of the cutting plane from the plane OD.

With $h = nB$, describe the circle $At_1 rB$; draw rd perpendicular to NB, then $nd = z$ at the point n. For $(nr)^2 - (nd)^2 = (rd)^2$, that is,

$$h^2 - z^2 = (rd)^2,\ \therefore\ rd = (h^2 - z^2)^{\frac{1}{2}}.$$

On account of the similarity of the triangles rnd and pnN, we have

$$Np : pn :: dr : rn, \text{ that is,}$$

$$f : 2b \cos^2 \frac{\pi u}{2c} :: (h^2 - z^2)^{\frac{1}{2}} : h;$$

from which we find

$$2b \cos^2 \frac{\pi u}{2c}\left(1 - \frac{z^2}{h^2}\right)^{\frac{1}{2}} = f,$$

the equation for all points of the curve formed by cutting the aft body by a plane whose distance from the sheer plane $= f$.

In Fig. XXXVII,—

$OX_1 = 98$ ft.; $OZ = nB = lB_1 = X_1D = 15$ ft.;

$Og = 28$ ft. $On = 34{\cdot}84$ ft.. $np = 14{\cdot}2055$ ft., $pN = p_1N_1 = m_2l = 5$ ft.; and

nd, the value of z at the point n, $= 14{\cdot}04$ ft. $Ol = 76{\cdot}3689$ ft.

At O the deck ordinate $Op_1 = 2b = 17{\cdot}5$ ft. is greater than $OZ = 15$ ft., at l the curve terminates where the deck ordinate $= f = 5$ ft.

THE AXIS ROUND WHICH A FLOATING BODY TURNS.

The imaginary line or axis on which a ship is supposed to oscillate, or round which she is supposed to turn, when she is acted upon by external forces, does not, except in very rare instances, pass through the centre of gravity of the floating mass. The exceptional instances, here alluded to, never have place in a properly designed and carefully ballasted ship; this we shall prove, after we have completed the geometry of Scott Russell's solid, out of which ships to suit different purposes may be constructed.

However, in the meantime, we shall take a simple case that does not require the application of the higher mathematics. When a force is required to turn a body, free to move, from a state of rest or equilibrium the centre of gravity of that body, so long as the force has to be continued, is being raised; and when a force is required to prevent a body from returning to a place of rest or equilibrium, the centre of gravity of that body is inclined to fall, reference being made to a fixed or stationary horizontal plane or line.

Let us take a solid homogeneous body of uniform shape and dimensions throughout the whole of its length, and suppose it to be placed upon a fluid of greater specific gravity than itself, in such a manner, that the centre of buoyant effort and the centre of gravity are in the same vertical line. And let it be required to determine the stability, when by the application of some external force, the body is deflected from the upright position, or from a position of equilibrium through a given angle.

Again, let the solid to which our present investigation refers be such, that the vertical transverse sections perpendicular to the (unknown) axis of motion, are equal and similar trapezoids, as indicated by *ACDE*, Figs. XXXIX to XLIII.

Reference being made to Fig. XXXIX, the solid floats on the surface of the water *KL* and *AEDC* is its position when in a state of equilibrium; *AKLC* being the portion of the vertical section above and out of the water, and *KLDE* the part immersed beneath the surface of the water. The point *G* is the centre of gravity of the whole section, the plane of which is supposed to pass through the centre of gravity of the body, and *g* is the centre of buoyancy, or the centre of gravity of the part immersed below the surface of the fluid; then since the body floats in a state of equilibrium, it follows, that *BF* the axis of the section, which passes through the points *G*, *g*, is perpendicular to *KL* in the line of flotation. As the investigation of the problem, here introduced, is designed so that it may be understood by those of ordinary mathematical skill, we deem it necessary to introduce the following geometrical proposition.—

PRELIMINARY PROPOSITION.

Let *KLDE*, Fig. XXXVIII, be a trapezoid; *KL* parallel to *ED*, and the angle *KED* equal to the angle *EDL*; it is required to draw from any point *M* in the side *KE*, a straight line *MN*, so that the area of the trapezium *MNDE* shall be equal to the area of the trapezoid *KLDE*. *Construction.* Draw *MP* parallel to *KL* or *ED*; produce *KE* and *LD* until they meet in *Q*; with *Q* as centre, and radius *QL*, describe the arc *SL*; from the point *P* draw *PS* perpendicular to *DL*, cutting the arc *SL* in *S*; then, the semicircle *QSN* passing through the points *Q*, *S*, and cutting *DL* produced in *N*, determines the point *N*; and the trapezium *MNDE* is equal to the trapezoid *KLDE*. See O. Byrne's "Vade-mecum de l'Ingénieur de Chemins de Fer," p. 121. Paris: De Napoléon Chaix, Rue Bergère.

The truth of this construction may be established thus :—

The area of the triangle $KQL = KQ \times QL \times \frac{1}{2} \sin KQL$; and the area of $MQN = MQ \times QN \times \frac{1}{2} \sin MQN$;

$$\therefore KQ \times QL = QL^2 = MQ \times QN = QP \times QN$$

$$\therefore QN = \frac{QL^2}{QP}.$$

Since, $QP \times QN = QL^2 = QS^2$, and PS perpendicular to QN; a right-angled triangle (right-angled at S) may be formed by joining the points Q, S, and S, N; and hence a semicircle may be made to pass through the points Q, S, and cut the line QN in N.

Let $a = HQ$; $b = KH = HL$; $c = KQ = QL$; and $n = KM$. Put $x = OL$; then $OK = 2b - x$. Draw MU and NV perpendicular to KOV. From similar triangles we have

$$KQ : QH :: KM : MU = \frac{an}{c}; \text{ and}$$

$$LQ : QH :: LN : NV = \frac{a \times LN}{c}.$$

It has been shown that $QN = \frac{c^2}{c - n}$, but

$$LN = QN - QL = \frac{c^2}{c - n} - c = \frac{cn}{c - n}.$$

$$\therefore NV = \frac{a}{c} \times \frac{cn}{c - n} = \frac{an}{c - n}.$$

Since the triangles KOM, NOL, have equal areas, $KO \times UM = OL \times NV$, or

$$(2b - x)\frac{an}{c} = x\frac{an}{c - n}; \ \therefore x = \frac{c - n}{c - \frac{1}{2}n} b;$$

x must always be less than b, for, the numerator of the fraction $\frac{c - n}{c - \frac{1}{2}n}$ is always less than the denominator.

If g and G be points taken in QH and QH produced, and perpendiculars gR, GI, drawn to NM, then, the four right-angled triangles, OUM; OHJ; GIJ; and gRJ are similar.

$$OK = 2b - x = \frac{c}{c - \frac{1}{2}n} b;$$

$$KU = \frac{nb}{c};$$

$$OU = \left(\frac{c}{c - \frac{1}{2}n} - \frac{n}{c}\right) b = \frac{(c - n)^2 + c^2}{2c - n}\left(\frac{b}{c}\right);$$

$$\text{and } OH = b - \frac{c - n}{c - \frac{1}{2}n}\, b = \frac{n}{2c - n}\, b.$$

$OU : UM :: OH : HJ$, that is

$$\frac{(c - n)^2 + c^2}{2c - n}\frac{b}{c} : \frac{an}{c} :: \frac{n}{2c - n}\, b : \frac{an^2}{(c - n)^2 + c^2} = HJ.$$

Putting $GH = p$ and $gH = q$, then $JG = p + \dfrac{an^2}{(c - n)^2 + c^2}$ and

$Jg = q - \dfrac{an^2}{(c - n)^2 + c^2}$. $\dfrac{UM}{OU}$ = tangent of the angle UOM,

that is, $\dfrac{an}{c}$ divided by $\dfrac{(c - n)^2 + c^2}{2c - n}\dfrac{b}{c}$, or $\dfrac{an}{b}\dfrac{2c - n}{(c - n)^2 + c^2}$ = tan UOM. Whence, all the angles of the right-angled triangles OUM, OHJ; GIJ, gRJ become known.

Twice the area of the triangle MNP divided by MN gives the perpendicular PT. Make $Mm = mN$, draw Pm; then, r is the centre of gravity of the triangle MPN, if $mr = \frac{1}{3}mP$. And the perpendicular $rt = \frac{1}{3}PT$. See O. Byrne's "Essential Elements of Practical Mechanics," p. 246.

$$MP^2 - PN^2 = MT^2 - TN^2 = (Mm + mT)^2 - (mN - mT)^2 = 4Mm \times mT;$$

$$\therefore mT = \frac{MP^2 - PN^2}{2MN}.$$

It is evident that the formulæ just adduced apply to all trapezoids and trapeziums circumstanced like $KLDE$ and $MNDE$, that is, to the many positions which may be occupied by the floating body while the water-line KL cuts the sides AE, DC; Figs. XXXIX and XL.

Now suppose, by the application of some horizontal external force f, the solid to revolve about an axis of motion until it comes unto the position shown in Fig. XL, in which state the equilibrium does not obtain.

Let BC = 12 ft. = AB; ED = 2·4 ft., EF = 1·2 ft. = FD, Fig. XXXIX; BH = 4·5; HF = 3·6, therefore, BF = 8·1; FQ = ·9 and BQ = 9. Consequently QC = 15 = AQ; and EQ = 1·5 = QD; AK = 7·5 = LC = KQ = QL.

$$KE = 6\cdot = LD.$$

From what has been shown, page 75, $BG = 2\frac{52}{55}$ ft.; G being the centre of gravity of the trapezoid $AEDC$; $GH = 1\frac{61}{110}$.

Again, $KH = 6\cdot = HL$, and $Hg = 1\frac{2}{5}$, g being the centre of gravity of the part, $KLDE$, immersed below the surface of the water; $Fg = 2\frac{1}{5}$.

Taking f_1, Fig. XXXIX, = the pressure of the water on the face KE and $f_3 =$ the pressure on the face DL; these forces f_1, f_3, acting on the levers f_1H and f_3H retain the body $AEDC$ in equilibrium and not the vertical force f_2.

Let a horizontal force f be applied to move the body and support it in the position shown in Fig. XL, so that $KM = 3$ ft.; then the position of the new water line MN is readily found by the preliminary proposition. $QN = 12\cdot 5$; $DN = QN - QD = 11$; MN will cross the line KL in O, and O is not in the centre of KL. G is further from the water line in the position Fig. XL, than in the position Fig. XXXIX. The point O being common to the two water lines, the body, Fig. XL has a tendency, if the equilibrium be disturbed, to turn round the point O, and not as is generally assumed, round the point G.

$$OH = \frac{n}{2c - n} b = 1\tfrac{1}{2}\text{; (see preliminary proposition)}$$

$$HJ = \frac{an^2}{(c-n)^2 + c^2} = \frac{9}{17};\quad OJ = \sqrt{\left(\frac{9}{17}\right)^2 + \left(\frac{3}{2}\right)^2} = \frac{15}{34}\sqrt{13}$$

$$\frac{9}{17}\text{ divided by }\frac{3}{2} = \frac{6}{17} = \cdot 3529412 = \tan. KOM;$$

$$\therefore KOM = 19^\circ\ 26'\ 24''\cdot 12.$$

$$\frac{9}{17}\text{ divided by }\frac{15\sqrt{13}}{34} = \frac{6\sqrt{13}}{65} = \cdot 3328201 = \sin KOM.$$

$$GJ = GH + HJ = 1\tfrac{61}{110} + \tfrac{9}{17} = \frac{433 \times 9}{110 \times 17};$$

$$gJ = gH - HJ = 1\tfrac{2}{5} - \tfrac{9}{17} = \frac{74}{5 \times 17};\text{ then,}$$

$OJ\ OH :: JG : GI$, that is,

$$\frac{15}{34}\sqrt{13} : \frac{3}{2} :: \frac{433 \times 9}{110 \times 17} : \frac{433 \times 9}{550\sqrt{13}} = 1\cdot 96515149.$$

$$\therefore GI = 1\cdot 96515\text{ is much greater than }GH = \frac{171}{110} = 1\cdot 5545.$$

$OJ : OH :: Jg : gR$, that is,

$$\frac{15\sqrt{13}}{34} : \frac{3}{2} :: \frac{74}{5 \times 17} : \frac{2{\cdot}96}{\sqrt{13}} = {\cdot}82096.$$

Consequently $gR = {\cdot}82396$ is less than $Hg = 1\frac{2}{5} = 1{\cdot}4$.

It leads to absurd conclusions to allow, or to suppose that the perpendicular distances of G and g from the water line remain constant; the form of the floating body, the distribution of its weights, or the inclining of the body, by ever so small an angle, do not effect, but, only modify the general principle.

MP, Fig. XL, $= 7{\cdot}2$; $ED, = 2{\cdot}4$; $nF = 1{\cdot}8$; then, if e be the centre of gravity of the trapezoid $MEDP$, $ne = \frac{3}{4}$ ft.

$$eJ = eH - HJ = en + nH - HJ = \frac{3}{4} + 1{\cdot}8 - \frac{9}{17} = \frac{687}{340}.$$

$OJ : OH :: eJ : ed$ the distance of the centre of gravity e, of the trapezoid $MEDP$, from the water line MN; that is,

$$\frac{15\sqrt{13}}{34} : \frac{3}{2} :: \frac{229 \times 3}{340} : \frac{6{\cdot}87}{\sqrt{13}} = ed.$$

$$MN = 4\sqrt{13};\ Mm = 2\sqrt{13} = mN.$$

$25{\cdot}92$ = area of $KEDL = MEDN$; $8{\cdot}64$ = area of $MPDE$; therefore, $25{\cdot}92 - 8{\cdot}64 = 17{\cdot}28$ = area of the triangle MPN.

$OJ : JH :: eJ : Jd$, or or $\dfrac{15\sqrt{13}}{34} : \dfrac{9}{17} :: \dfrac{229 \times 3}{340} : \dfrac{41{\cdot}22}{17\sqrt{13}} = Jd.$

$$PT = \frac{17{\cdot}28 \times 2}{4\sqrt{13}} = \frac{8{\cdot}64}{\sqrt{13}};\ \text{and } rt = \frac{2{\cdot}88}{\sqrt{13}}.$$

$QN = 12{\cdot}5$; $QD = 1{\cdot}5$; $QP = 4{\cdot}5$; $PN = 8$; therefore, as

$$\frac{PN^2 - MP^2}{2MN} = mT,\ mT = \frac{8^2 - (7{\cdot}2)^2}{8\sqrt{13}} = \frac{1{\cdot}52}{\sqrt{13}},\ \text{and } mt = \frac{1{\cdot}52}{3\sqrt{13}}.$$

If Y be the centre of gravity of the trapezium $MEDN$, then

$$YZ \times \text{area } MEDN = ed \times \text{area } MEDP + rt \times \text{area } MPN$$

$$YZ = \frac{\dfrac{6{\cdot}87}{\sqrt{13}} \times 8{\cdot}64 + \dfrac{2{\cdot}88}{\sqrt{13}} \times 17{\cdot}28}{25{\cdot}92} = \frac{4{\cdot}21}{\sqrt{13}}.$$

$$ON = \frac{5\sqrt{13}}{2},\ OM = \frac{3\sqrt{13}}{2}.$$

$$ON - mN = Om;\ \frac{5\sqrt{13}}{2} - 2\sqrt{13} = \frac{\sqrt{13}}{2} = Om;$$

$$Om - mt = Ot;\ \frac{\sqrt{13}}{2} - \frac{1{\cdot}52}{3\sqrt{13}} = \frac{17{\cdot}98\sqrt{13}}{39} = ot.$$

Hence, the perpendicular distance, IZ, between the directions of the downward pressure through G and the upward pressure through Y, may be accurately determined for all floating bodies of the form under consideration, by plane geometry, and for all inclinations that the body may assume in being moved from the position shown in Fig. XXXIX, to the position shown in Fig. XLI.

The variable distance *IZ* (the equilibrating lever) is improperly termed a lever, it has no existence when the floating body is in the positions shown in Figs. XXXIX and XLIII; nor has this so-called lever an existence in an intermediate position between that of Fig. XLII and that of Fig. XLIII.

The point *G* cannot he taken as a prop for the variable lever *IZ*, for, as the body is turned by the applied forces, *G* continually changes its distance from the water line which remains fixed.

Neither the *principle of living forces*, nor the *principle of D'Alembert*, can be applied to determine the circumstances of the motion of the floating body *AEDC* by merely involving those of the pressures through *G* and *Y* (Figs. XXXIX to XLIII), on the variable line termed the *equilibrating lever*, which is without a fixed point or line of support.

Monsieur S. D. Poisson, a profound mathematician, in his apparently exhaustive treatise on mechanics, evades the solution of this problem, and takes shelter in the obscurity of symbols of operation in his endeavour to apply the *principle of living forces.*

Now, if we consider the moments of the applied forces f, f_1, f_2, &c., and the downward pressure through *G*, with respect to the point *O*, or a line that remains fixed, around which the body actually turns; the principle of living forces, or the principle of D'Alembert (which may be regarded as a dynamical axiom), may be applied to determine the circumstances of the motion of the body *AEDC*. In the investigation before us, the principle of D'Alembert is more readily applied than the principle of living forces as the point *O*, which neither rises nor falls, is constrained to remain in the stationary or fixed line, the water line. Nor does it affect the application of the dynamical axiom of

D'Alembert to suppose H, Fig. XXXIX; O, Fig. XL; O, Fig. XLI; O, Fig. XLII; and Z, Fig. XLIII, to be one and the same point. The form of the floating body and particular states of its immersion might render it necessary or convenient to take the point O, in a stationary line parallel to the water line, but, at a given distance from it, in such cases the dynamical axiom of D'Alembert is also applicable.

Referring to Fig. XLI, in which the side DC is parallel to the water line MN; and putting $W =$ the whole weight of the body, $AEDC$, concentrated in its centre of gravity G; then, the body has no tendency to turn out of this position, when

$$W \times OI + f_3 \times f_3O = f \times fO + f_1 \times f_1O + f_2 \times f_2O + f_4 \times f_4O;$$

and as long as this equality exists the body may be moved from place to place without disturbing the position of the point O or of the water line MN. But general equations of equilibrium for every position into which the body may be forced can be determined by the *principle of virtual velocities*, since, the lengths and directions of all the required lines, and the exact positions of all the required points may be found, in the family of solids under consideration, by plane geometry; in all cases the predicated results may be verified by experiments.

The principal of virtual velocities shows that the hody $ACDE$ will rest in the position, Fig. XLII, when

$$W \times OI + f \times fO + f_2 \times f_2O = f_1 \times f_1O.$$

From this equation it will be found that f becomes negative, that is, the force f has to applied in an opposite direction to retain the body in equilibrium with the line or face AC perpendicular to the water line MN.

If the body, Fig. XLII, be moved until GI and YZ be in the same straight line, perpendicular to the water line, the principle of virtual velocities shows that no value whatever can be given to f that will retain the body at rest in this position, although the downward pressure through G and the upward pressure through Y are in the same straight line which is perpendicular to the water line. A contrary erroneous result would be arrived at, if we had continued to reason with respect to the forces acting through G and Y and the *equilibrating lever* which vanishes when GI and YZ are in the same vertical line.

In Fig. XLIII, the face AC is parallel to the water line MN, the pressures through Y and G are in the same vertical line YG, and the *equilibrating lever* vanishes; but it is not the pressure f_2 acting through Y and the pressure of W acting in a contrary direction through G that keep the body at rest in this position, but the equality of the moments $f_1 \times f_1Z$ and $f_3 \times f_3Z$.

We have taken a solid homogeneous floating body of uniform shape and dimensions throughout the whole of its length; and for all such solids we have shown how the lines and points required in the investigation may be accurately found by plane geometry. And we have shown that it is absurd to attempt to apply the *principle of living forces* or the *principle of D'Alembert* to determine the circumstances appertaining to the turning motion of a floating body, by merely taking into account the downward pressure of the floating body, the upward pressure of the displaced water, and the perpendicular distance of the directions of these pressures, which distance has been termed the *equilibrating lever.*

We have also proved, by the *principle of virtual velocities,* that it is absurd to seek for the conditions under which the floating body would assume a state of rest or equilibrium through the action of those pressures and the *equilibrating lever* which is without prop or support.

On the other hand, we have proved mathematically that the external pressures and the downward pressure of the body may be combined, according to the principle of D'Alembert, to determine the nature and circumstances of the motions of the floating body. And further we have established, by the *principle of virtual velocities,* the conditions under which the body assumes a state of rest or of equilibrium in any given position by the action of the external forces and the downward pressure of the body.

The absurdities that we have pointed out cannot be cleared away by altering the shape or the dimensions of the floating body; and it requires but little reflection to perceive, that the distribution of weights inside the floating body, though all changes in their position must be allowed for, do not affect the general process of reasoning by which these absurdities have been pointed out.

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Fig. IVc.

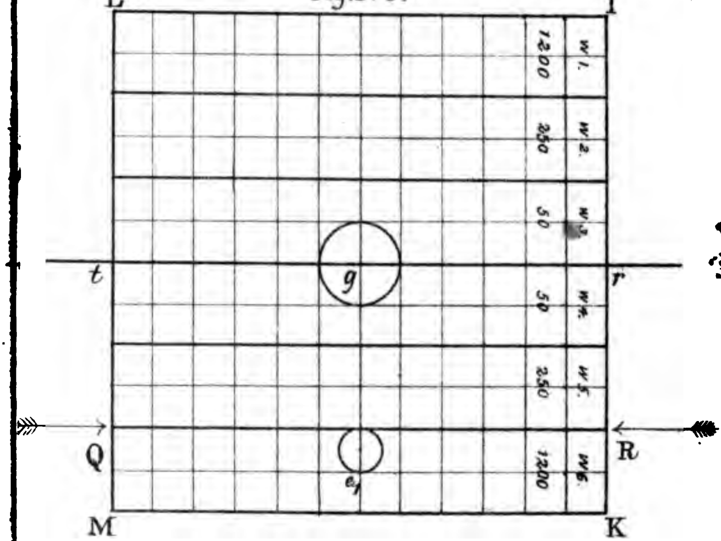


Fig. Vc.

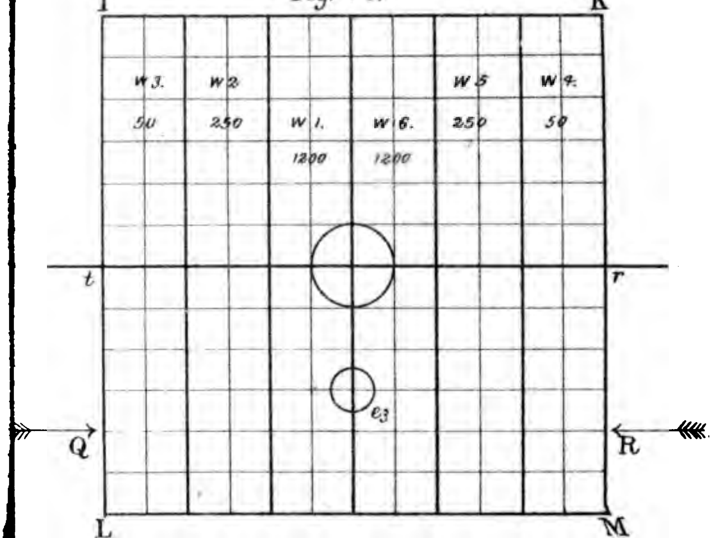
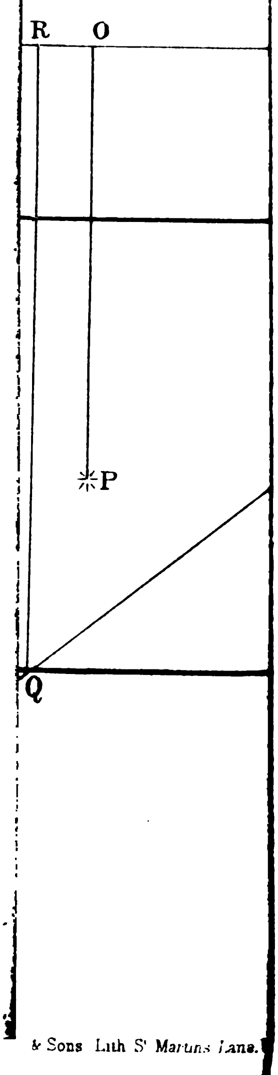


Fig. XII.



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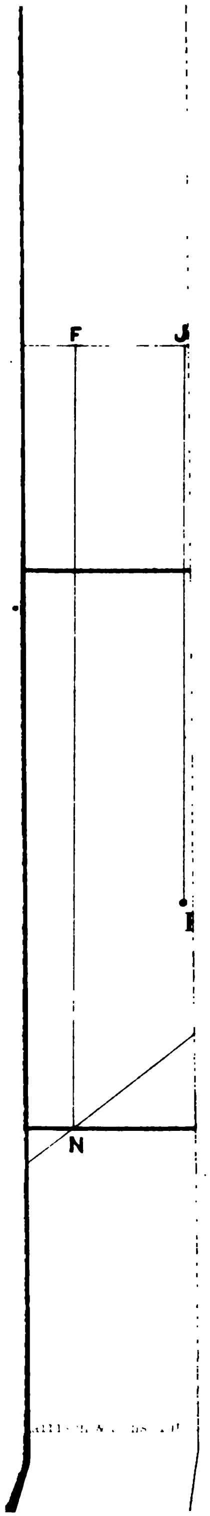
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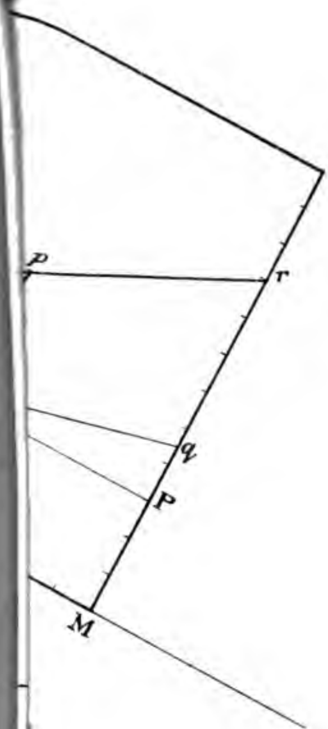
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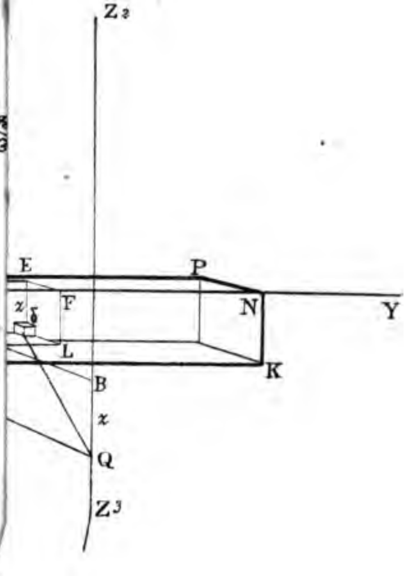
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Fig. XVIII.

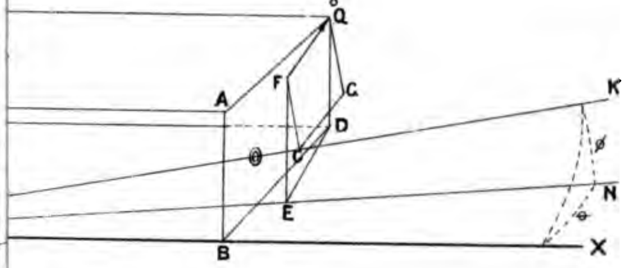


Fig. XIX.

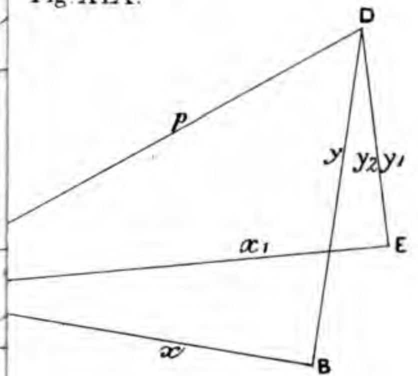
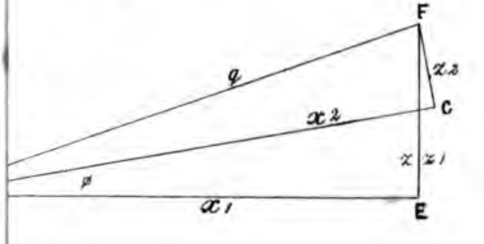


Fig. XX.



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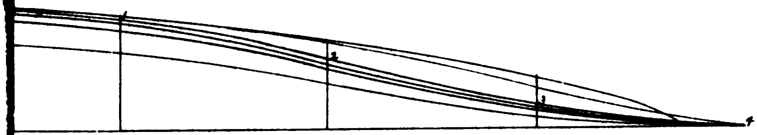

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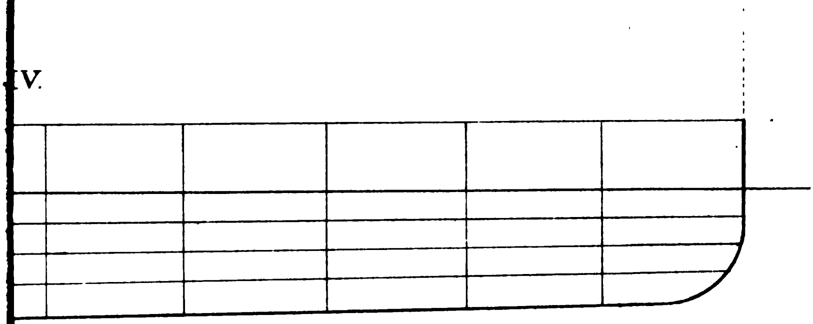

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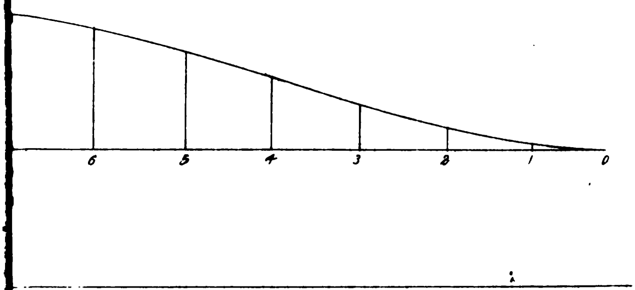

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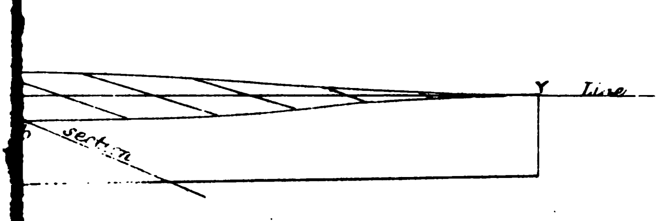

Fig. X

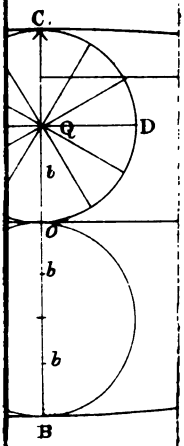

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Fig. XXX.

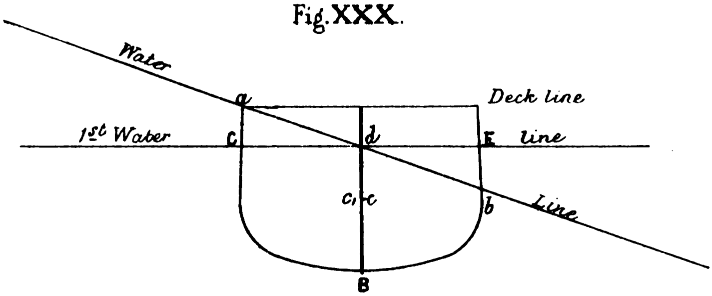


Fig. XXXI.

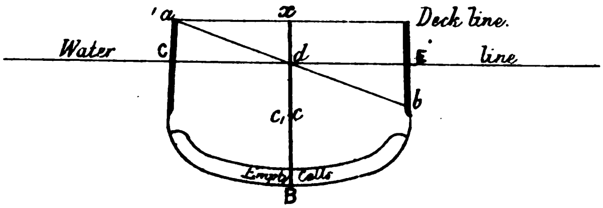
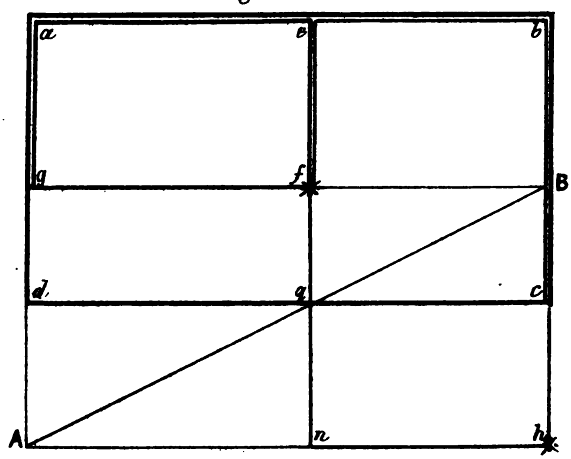


Fig. XXXII.



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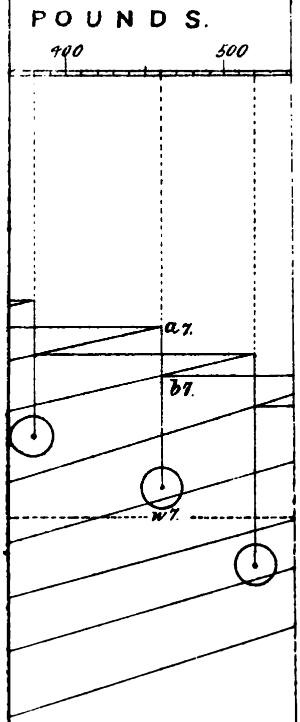

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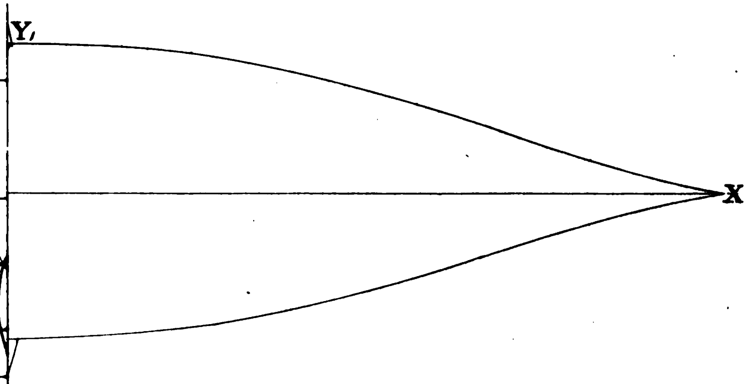
XXVI.



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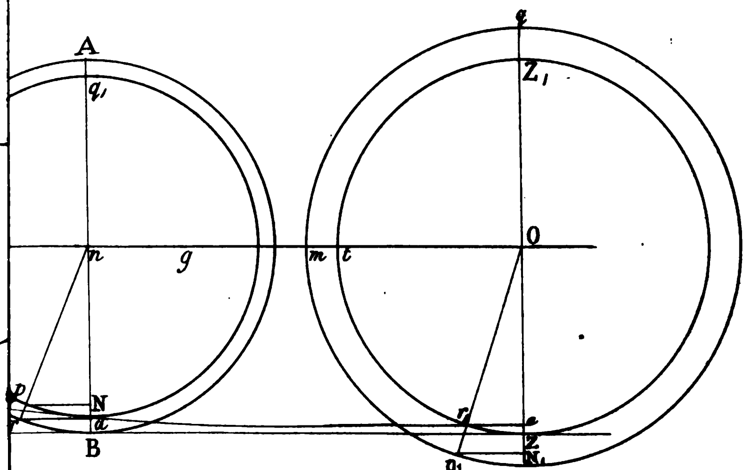
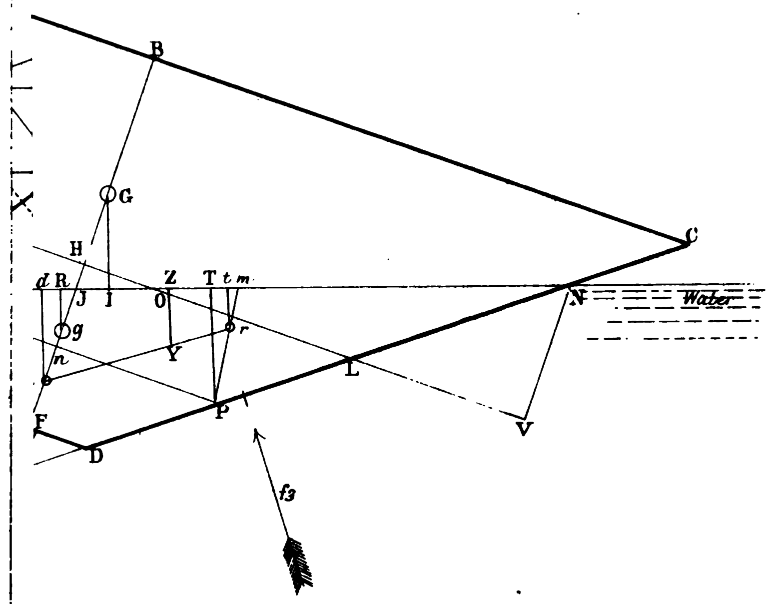
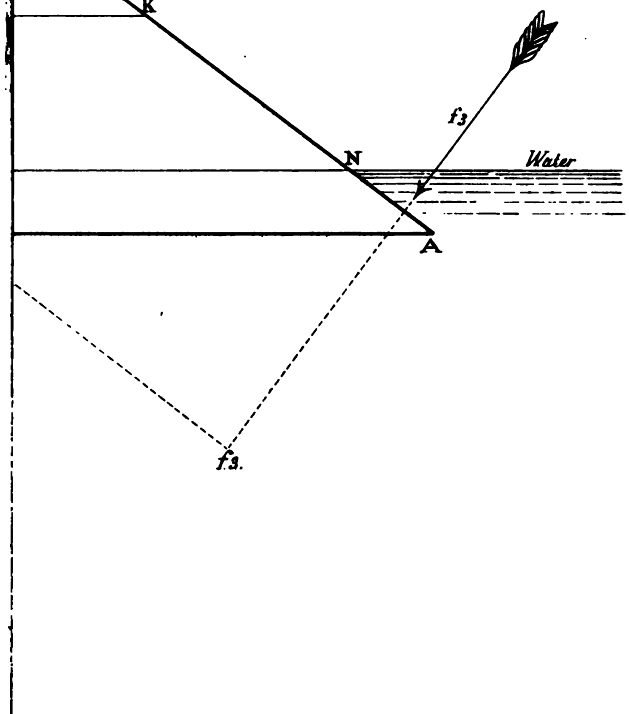


Fig. XL.

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